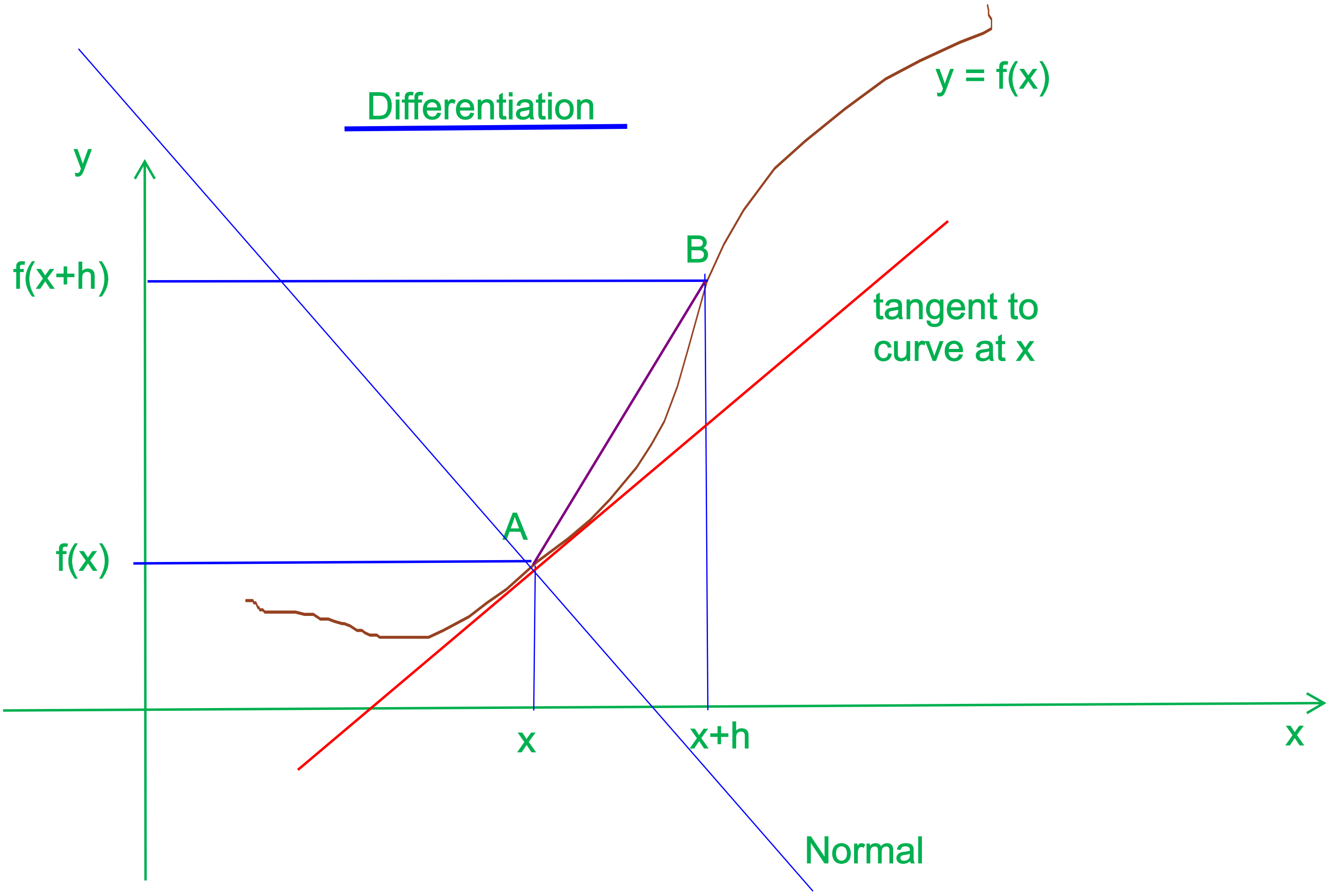


$$gradient = \frac{\Delta y}{\Delta x} = \frac{f(x+h)-f(x)}{h} \quad (gradient\ of\ the\ line\ AB)$$

$$gradient\ of\ \text{tangent} = \lim_{h \to 0}\left[\frac{f(x+h)-f(x)}{h}\right] \quad (\text{red line})$$

differentiation from first principles

$$\frac{dy}{dx} = \lim_{h \to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$y = f(x) = x^3$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{(x+h)^3 - x^3}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{x^3 + 3x^2h + 3xh^2 + h^3}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{3x^2h + 3xh^2 + h^3}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[3x^2 + 3xh + h^2\right]$$

$$\frac{dy}{dx} = 3x^2$$

$$y = 5x^4$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{5(x+h)^4 - 5x^4}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{5\left(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4\right) - 5x^4}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{5x^4 + 20x^3h + 30x^2h^2 + 20xh^3 + 5h^4 - 5x^4}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{20x^3h + 30x^2h^2 + 20xh^3 + 5h^4}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[20x^3 + 30x^2h + 20xh^2 + 5h^3\right]$$

$$\frac{dy}{dx} = 20x^3 \ (gradient\ function)$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right]$$

$y = f(x) = x^n$ where n is any integer

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{(x+h)^n - x^n}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{x^n + \binom{n}{1}x^{n-1}h + \binom{n}{2}x^{n-2}h^2 + .,.......... + h^n - x^n}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\frac{\binom{n}{1}x^{n-1}h + \binom{n}{2}x^{n-2}h^2 + .,.......... + h^n}{h}\right]$$

$$\frac{dy}{dx} = \lim_{h\to 0}\left[\binom{n}{1}x^{n-1} + \binom{n}{2}x^{n-2}h + .,.......... + h^{n-1}\right]$$

$$\frac{dy}{dx} = nx^{n-1}$$

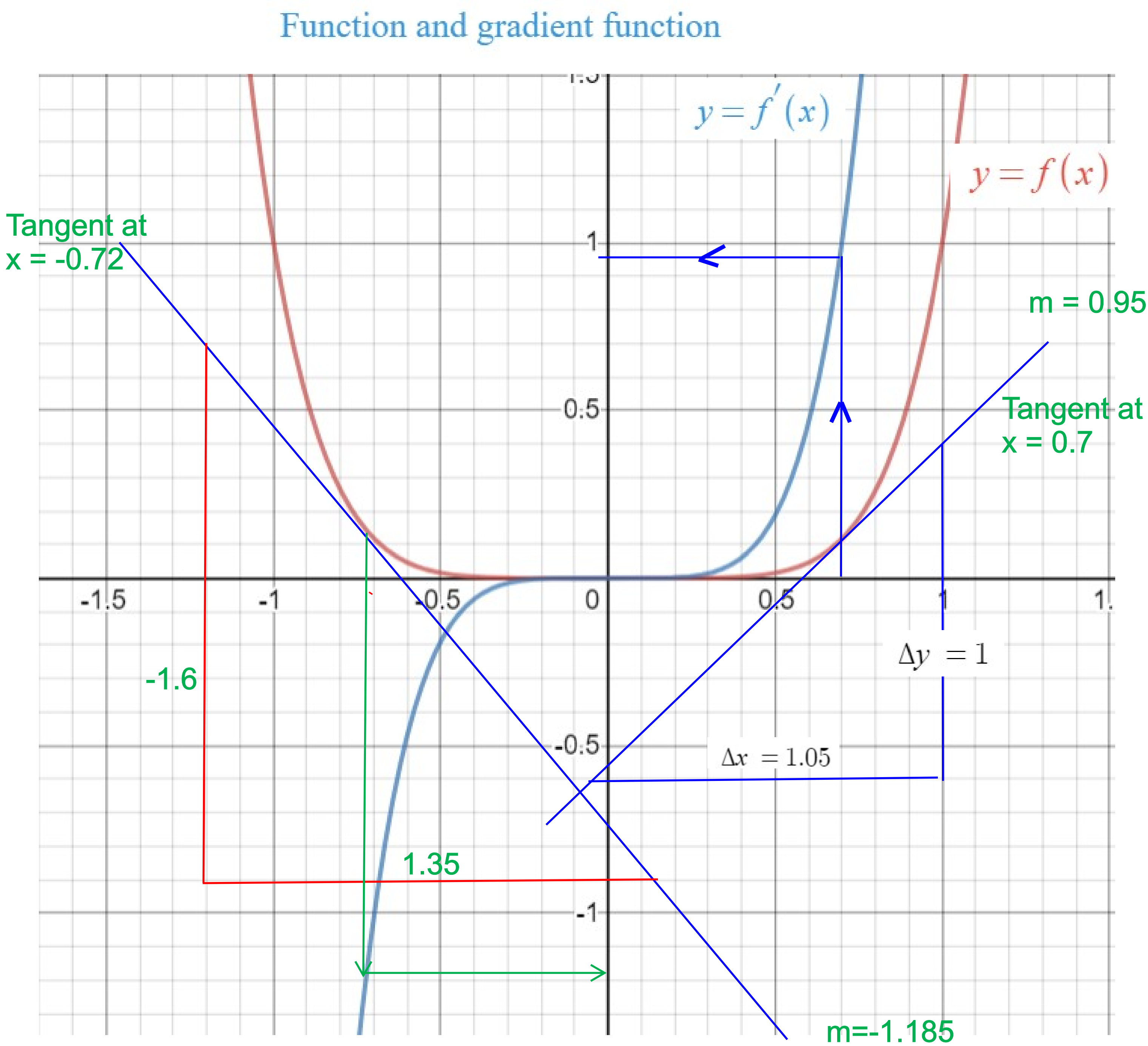
$$\frac{dy}{dx} = nx^{n-1}$$

$y = x^n \Rightarrow \dfrac{dy}{dx} = nx^{n-1}$ generalising

$y = x^6 \Rightarrow \dfrac{dy}{dx} = 6x^5$ gradient function

$y = 4x^7 \Rightarrow \dfrac{dy}{dx} = 28x^6$

$y = 3x^5 + x^2 \Rightarrow \dfrac{dy}{dx} = 15x^4 + 2x$



Function and gradient function
$y = f'(x)$
$y = f(x)$
Tangent at
x = -0.72
m = 0.95
Tangent at
x = 0.7
-1.6
$\Delta y = 1$
$\Delta x = 1.05$
1.35
m=-1.185

Solomon WSA
differentiation

Q2

b) $y = x + x^3 \Rightarrow \dfrac{dy}{dx} = 1 + 3x^2$

c) $y = x^4 + 2 \Rightarrow \dfrac{dy}{dx} = 4x^3$

f) $y = x^2 - 4x + 1 \Rightarrow \dfrac{dy}{dx} = 2x - 4$

g) $y = x^{-1} - x^{-5} \Rightarrow \dfrac{dy}{dx} = -x^{-2} + 5x^{-6}$

Q4

b) $f(x) = x^{\frac{1}{2}} - 5 \quad \Rightarrow \quad f'(x) = \dfrac{1}{2}x^{-\frac{1}{2}}$

f) $f(x) = 2x^{\frac{1}{6}} + x^{\frac{1}{4}} \Rightarrow f'(x) = \dfrac{1}{3}x^{-\frac{5}{6}} + \dfrac{1}{4}x$

c) $f(x) = x + 4x^{\frac{1}{2}} \quad \Rightarrow \quad f'(x) = 1 + 2x^{-\frac{1}{2}}$

g) $f(x) = 3x^{-1} - 5x^{-1} \quad \Rightarrow \quad f'(x) = -3x^{-2} + 5x^{-2}$

Q5

b) $y = 4 - \dfrac{1}{x}$

$y = 4 - x^{-1} \Rightarrow \dfrac{dy}{dx} = x^{-2}$

c) $y = 3x^2 + \sqrt[3]{x}$

$y = 3x^2 + x^{\frac{1}{3}} \quad \Rightarrow \dfrac{dy}{dx} = 6x + \dfrac{1}{3}x^{-\frac{2}{3}}$

f) $y = \dfrac{6}{\sqrt[4]{x}} = 6x^{-\frac{1}{4}} \quad \Rightarrow \dfrac{dy}{dx} = -\dfrac{3}{2}x^{-\frac{5}{4}}$

g) $y = \sqrt{x^5} = x^{\frac{5}{2}} \Rightarrow \dfrac{dy}{dx} = \dfrac{5}{2}x^{\frac{3}{2}}$

Q7

c) $y = \dfrac{4x^3 + x}{x^2} \quad \Rightarrow y = \dfrac{4x^3}{x^2} + \dfrac{x}{x^2} = 4x + x^{-1} \quad \Rightarrow \dfrac{dy}{dx} = 4 - x^{-2}$

g) $y = \dfrac{9x - 2}{3x} \quad \Rightarrow y = \dfrac{9x}{3x} - \dfrac{2}{3x} = 3 - \dfrac{2}{3}x^{-1} \quad \Rightarrow \dfrac{dy}{dx} = \dfrac{2}{3}x^{-2}$

Q8

f) $y = 6x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

first differential $\dfrac{dy}{dx} = 3x^{-\frac{1}{2}} + \dfrac{1}{2}x^{-\frac{3}{2}}$

second differental $\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left(\dfrac{dy}{dx}\right) = -\dfrac{3}{2}x^{-\frac{3}{2}} - \dfrac{3}{4}x^{-\frac{5}{2}}$

WSC
Q2

d) $y = x^2 - 2x^{-1}$ $(2,3)$

$$\frac{dy}{dx} = 2x + 2x^{-2}$$

$$x = 2 \quad \frac{dy}{dx} = 2(2) + 2(2)^{-2} = 4 + \frac{1}{2} = \frac{9}{2}$$

5 For the curve with equation $y = 2x^2 - 5x + 1$,

a find $\frac{dy}{dx}$,

b find the value of x for which $\frac{dy}{dx} = 7$.

$$y = 2x^2 - 5x + 1$$

a) $\frac{dy}{dx} = 4x - 5$

b) when $\frac{dy}{dx} = 7 \quad \Rightarrow 4x - 5 = 7$

$implies \quad x = 3$

equation of tangent

Q8d) $y = x^3 - 4x^2 + 2$ $(3,-7)$

$$\frac{dy}{dx} = 3x^2 - 8x$$

$$when\ x = 3 \quad \Rightarrow \quad \frac{dy}{dx} = 3(3)^2 - 8(3) = 27 - 24 = 3$$

$m = 3$ $point$ $(3,-7)$

$$y - y_1 = m(x - x_1)$$

$$y + 7 = 3(x - 3)$$

9 Find an equation of the tangent to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

b) $y = x - 3\sqrt{x}$ $\quad (4,-2)$

$$y = x - 3x^{\frac{1}{2}}$$

$$\frac{dy}{dx} = 1 - \frac{3}{2}x^{-\frac{1}{2}}$$

$$when\ x = 4 \Rightarrow \frac{dy}{dx} = 1 - \frac{3}{2}(4)^{-\frac{1}{2}} = 1 - \frac{3}{4} = \frac{1}{4}$$

$$m_t = \frac{1}{4} \qquad (4,-2)$$

$$y - y_1 = m(x - x_1)$$

$$y + 2 = \frac{1}{4}(x - 4)$$

$$4y + 8 = x - 4$$

$$4y - x + 12 = 0$$

10 Find an equation of the normal to each curve at the given point. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

$$y = 3x^2 + 7x + 7 \qquad (-2,5)$$

$$\frac{dy}{dx} = 6x + 7$$

$$when\ x = -2 \ \Rightarrow \frac{dy}{dx} = 6(-2) + 7 = -5$$

$$m_t = -5 \Rightarrow m_n = \frac{1}{5} \quad (-2,5)$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{1}{5}(x + 2)$$

$$5y - 25 = x + 2$$

$$x - 5y + 27 = 0$$

Equation of Normal to curve

10d) $y = x - \frac{6}{x} = x - 6x^{-1}$ $(3,1)$

$\frac{dy}{dx} = 1 + 6x^{-2}$

$when\, x = 3 \Rightarrow \frac{dy}{dx} = 1 + \frac{6}{3^2} = 1 + \frac{6}{9} = 1 + \frac{2}{3} = \frac{5}{3}$

$m_t = \frac{5}{3}$ $implies$ $m_n = -\frac{3}{5}$ $(3,1)$

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{3}{5}(x - 3)$

$5y - 5 = -3x + 9$

$3x + 5y - 14 = 0$

Q15 The curve C has the equation $y = 6 + x - x^2$.

a Find the coordinates of the point P, where C crosses the positive x-axis, and the point Q, where C crosses the y-axis.

b Find an equation of the tangent to C at P.

c Find the coordinates of the point where the tangent to C at P meets the tangent to C at Q.

Q15 $y = 6 + x - x^2$

a) $At\, P\; y = 0 \Rightarrow 6 + x - x^2 = 0$

$x^2 - x - 6 = 0$

$(x - 3)(x + 2) = 0$

$x = 3$ or $x = -2$

$we\ require\ x > 0 \Rightarrow x = 3$ $P(3,0)$

$At\ Q;\ x = 0 \Rightarrow y = 6$ $Q(0,6)$

b) Equation of tangent

$\frac{dy}{dx} = 1 - 2x$

$At\, P\; x = 3 \Rightarrow \frac{dy}{dx} = 1 - 2(3) = -5$

$m = -5$ $P(3,0)$

$y - y_1 = m(x - x_1)$

$y = -5(x - 3)$

c) Equation of tangent $At\ Q$

$\frac{dy}{dx} = 1 - 2x$

$At\, Q\; x = 0 \Rightarrow \frac{dy}{dx} = 1 - 2(0) = 1$

$m = 1$ $Q(0,6)$

$y = x + 6$

solve simultaneously with $y = -5(x - 3)$

$\Rightarrow x + 6 = -5(x - 3)$

$x + 6 = -5x + 15$

$6x = 9$

$x = \frac{3}{2}$

$when\, x = \frac{3}{2}$ $y = \frac{3}{2} + 6 = \frac{15}{2}$

point of intersection $\left(\frac{3}{2}, \frac{15}{2}\right)$

17 The line with equation $y = 2x + k$ is a normal to the curve with equation $y = \frac{16}{x^2}$.

Find the value of the constant k.

$$y = \frac{16}{x^2} = 16x^{-2}$$

$$\frac{dy}{dx} = -32x^{-3}$$

given that $y = 2x + k$ *is normal to the curve*

$$\frac{dy}{dx} = -\frac{1}{2} = -32x^{-3}$$

$$x^3 = 64 \ \Rightarrow x = 4$$

$$y = \frac{16}{x^2}$$

$$\textit{when } x = 4 \ \Rightarrow y = \frac{16}{4^2} = 1$$

$$(4,1)$$

sub into $y = 2x + k$

$$\Rightarrow 1 = 2(4) + k \ \Rightarrow k = -7$$

19 Water is poured into a vase such that the depth, h cm, of the water in the vase after t seconds is given by $h = kt^{\frac{1}{3}}$, where k is a constant. Given that when $t = 1$, the depth of the water in the vase is increasing at the rate of 3 cm per second,

a find the value of k,

b find the rate at which h is increasing when $t = 8$.

$$h = kt^{\frac{1}{3}}$$

$$\frac{dh}{dt} = \frac{k}{3}t^{-\frac{2}{3}}$$

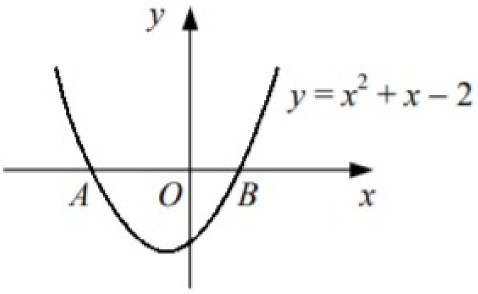
$$\frac{dh}{dt} = 3cm/\mathrm{s} \ \textit{when } t = \ 1$$

$$3 = \frac{k}{3}(1)^{-\frac{2}{3}} \quad \textit{implies } k = 9$$

$$\therefore \frac{dh}{dt} = 9t^{-\frac{2}{3}}$$

$$\textit{when } t \ = \ 8 \Rightarrow \frac{dh}{dt} = 9(8)^{-\frac{2}{3}} = 9 \times \frac{1}{4} = \frac{9}{4}\, cm/\mathrm{s}$$

3



The diagram shows the curve $y = x^2 + x - 2$. The curve crosses the x-axis at the points A $(a, 0)$ and B $(b, 0)$ where $a < b$.

a Find the values of a and b. **(3**

b Show that the normal to the curve at A has the equation

$$x - 3y + 2 = 0.$$ **(5**

The tangent to the curve at B meets the normal to the curve at A at the point C.

c Find the exact coordinates of C. **(4**

a) $y = x^2 + x - 2$

crosses x-axis when y=0

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = 1 \text{ or } -2$

$a = -2, b = 1$

b) $\frac{dy}{dx} = 2x + 1$

$at\ A\ x = -2 \Rightarrow \frac{dy}{dx} = 2(-2) + 1 = -3$

$m_t = -3 \Rightarrow m_n = \frac{1}{3}\quad A(-2,0)$

$y - y_1 = m(x - x_1)$

$y = \frac{1}{3}(x+2)$

$3y = x + 2$

$x - 3y + 2 = 0$

c) $\frac{dy}{dx} = 2x + 1$

$at\ B\ x = 1 \Rightarrow \frac{dy}{dx} = 2(1) + 1 = 3$

$m_t = 3\quad B(1,0)$

$y - y_1 = m(x - x_1)$

$y = 3(x-1)$

$y = 3x - 3$

$y = 3x - 3$

$x - 3y + 2 = 0$

solve simultaneously

$x - 3(3x-3) + 2 = 0$

$x - 9x + 9 + 2 = 0$

$8x = 11$

$x = \frac{11}{8} \Rightarrow y = \frac{9}{8}$

$C\left(\frac{11}{8}, \frac{9}{8}\right)$

4 Given that $y = \dfrac{x^2 - 6x - 3}{3x^{\frac{1}{2}}}$, show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can be expressed in the form $\dfrac{(x+a)^2}{bx^{\frac{3}{2}}}$, where a and b are integers to be found. **(6)**

$$y = \frac{x^2 - 6x - 3}{3x^{\frac{1}{2}}} = \frac{x^2}{3x^{\frac{1}{2}}} - \frac{6x}{3x^{\frac{1}{2}}} - \frac{3}{3x^{\frac{1}{2}}} = \frac{1}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} - x^{-\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}} = \frac{1}{2}x^{-\frac{3}{2}}\left(x^2 - 2x + 1\right) = \frac{(x-1)^2}{2x^{\frac{3}{2}}}$$

9 $$y = x^2 + 3x^{\frac{1}{2}}.$$

a Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. **(2)**

b Show that $2x\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + \dfrac{\mathrm{d}y}{\mathrm{d}x} - 6x = 0$. **(4)**

a) $y = x^2 + 3x^{\frac{1}{2}}$

$$\frac{dy}{dx} = 2x + \frac{3}{2}x^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = 2 - \frac{3}{4}x^{-\frac{3}{2}}$$

b) $2x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 6x = 0$

$$2x\left(2 - \frac{3}{4}x^{-\frac{3}{2}}\right) + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$$

$$4x - \frac{3}{2}x^{-\frac{1}{2}} + 2x + \frac{3}{2}x^{-\frac{1}{2}} - 6x$$

$$\cancel{6x} - \cancel{6x} - \cancel{\frac{3}{2}x^{-\frac{1}{2}}} + \cancel{\frac{3}{2}x^{-\frac{1}{2}}} = 0$$

12 A curve has the equation $y = \sqrt{x}\,(k - x)$, where k is a constant.

Given that the gradient of the curve is $\sqrt{2}$ at the point P where $x = 2$,

a find the value of k, **(5)**

b show that the normal to the curve at P has the equation

$$x + \sqrt{2}\,y = c,$$

where c is an integer to be found. **(5)**

a) $y = \sqrt{x}\,(k - x) = kx^{\frac{1}{2}} - x^{\frac{3}{2}}$

$$\frac{dy}{dx} = \frac{k}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$$

$$when\; x = 2 \Rightarrow \frac{dy}{dx} = \sqrt{2}$$

$$\sqrt{2} = \frac{k}{2} \times 2^{-\frac{1}{2}} - \frac{3}{2} \times 2^{\frac{1}{2}}$$

$$\sqrt{2} = \frac{k}{2} \times \frac{1}{\sqrt{2}} - \frac{3}{2} \times \sqrt{2}$$

$$\sqrt{2} = \frac{k}{2} \times \frac{\sqrt{2}}{2} - \frac{3}{2} \times \sqrt{2}$$

$$1 = \frac{k}{4} - \frac{3}{2}$$

$$\frac{5}{2} = \frac{k}{4} \;\Rightarrow\; k = 10$$

b) $y = \sqrt{x}\,(k - x) = kx^{\frac{1}{2}} - x^{\frac{3}{2}}$

$$k = 10$$

$$y = \sqrt{x}\,(10 - x)$$

$$when\; x = 2 \;\Rightarrow y = \sqrt{2}\,(10 - 2) = 8\sqrt{2}$$

$$\left(2, 8\sqrt{2}\right) \quad m_t = \sqrt{2} \;\Rightarrow\; m_n = -\frac{1}{\sqrt{2}}$$

$$y - y_1 = m\left(x - x_1\right)$$

$$y - 8\sqrt{2} = -\frac{1}{\sqrt{2}}\,(x - 2)$$

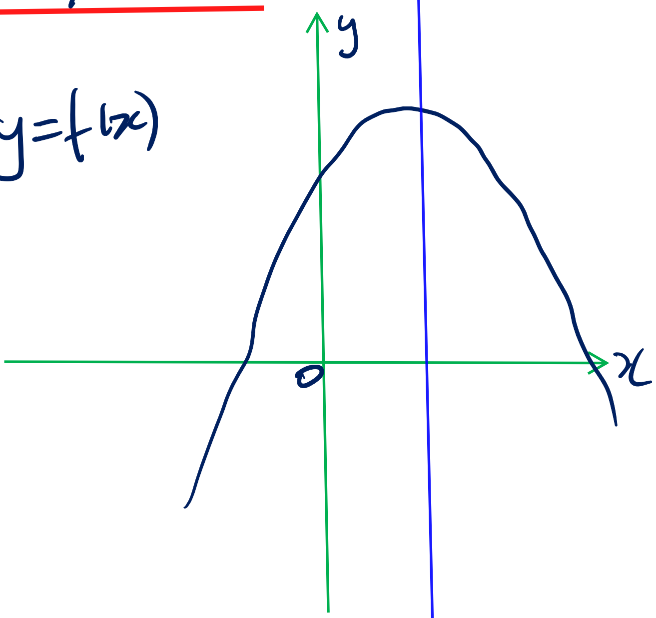
$$\sqrt{2}\,y - 16 = -x + 2$$

$$x + \sqrt{2}\,y = 18$$

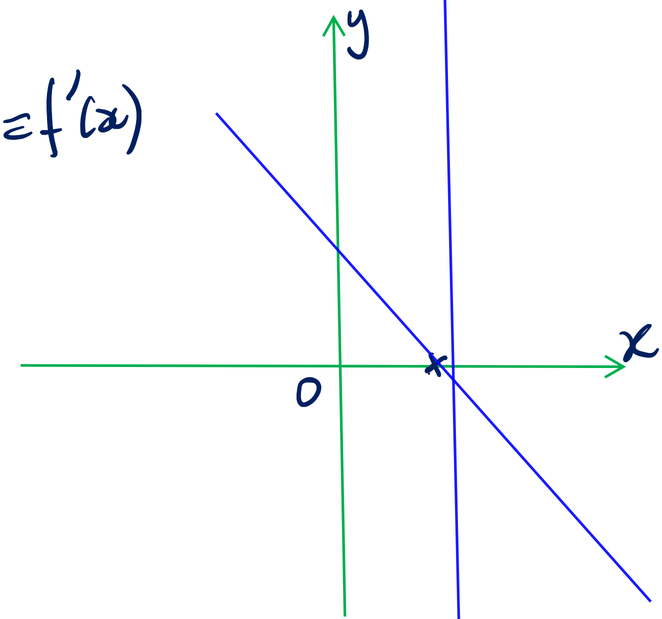
Differentiation and Nature of turning points

Maximum

$y=f(x)$

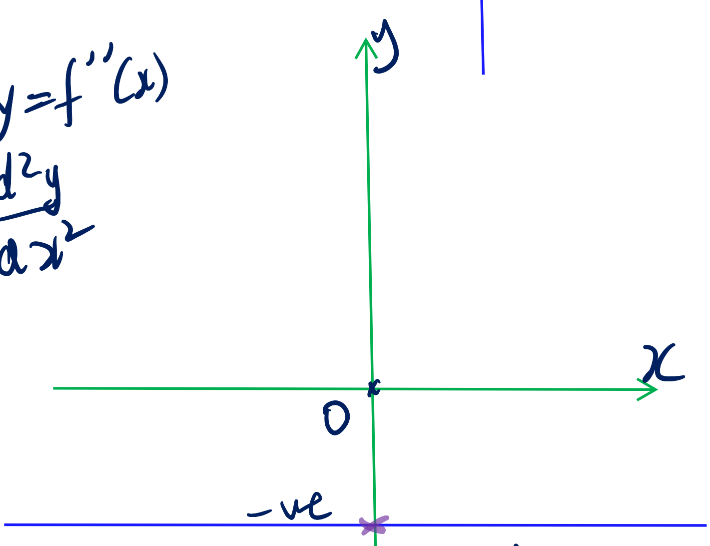


$y=f'(x)$



$y=f''(x)$

$\frac{d^2y}{dx^2}$

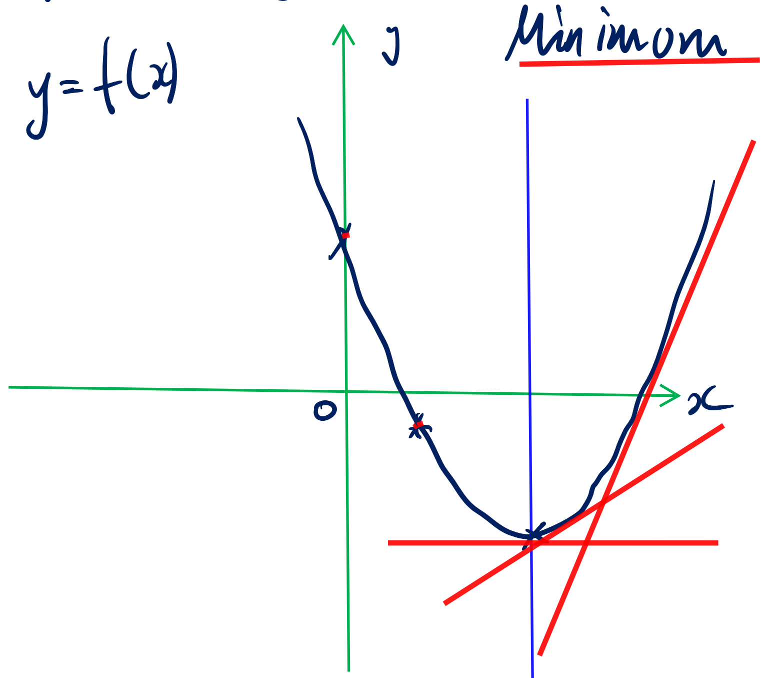


Condition for a Maximum

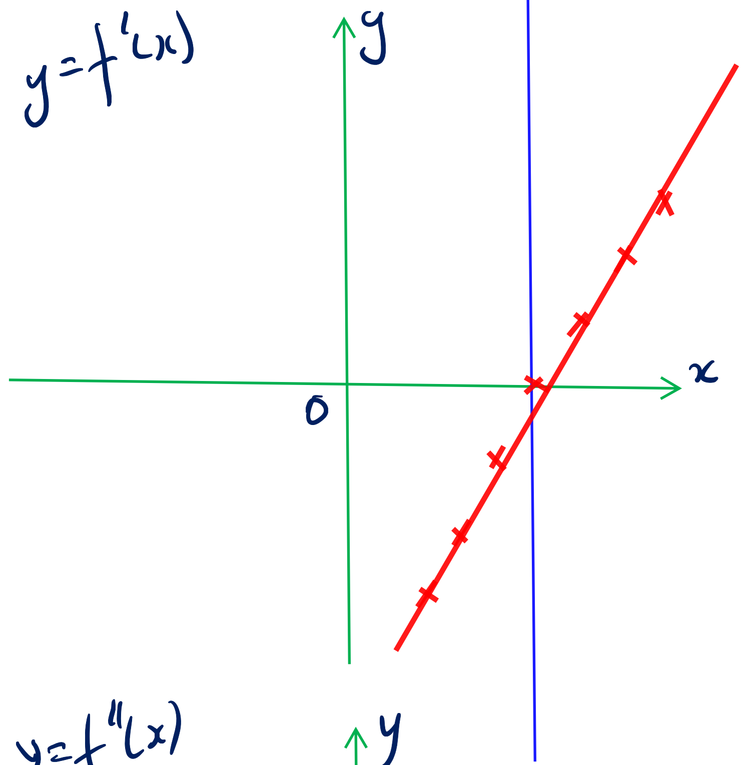
$\frac{d^2y}{dx^2} < 0$ (Negative)

Minimum

$y=f(x)$

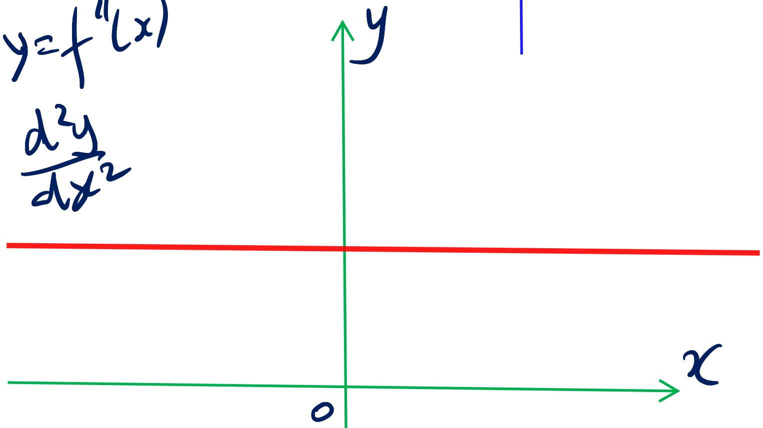


$y=f'(x)$



$y=f''(x)$

$\frac{d^2y}{dx^2}$



Condition for a Minimum

$\frac{d^2y}{dx^2} > 0$ (+ve).

Exam Questions, Differentiation part 1.

(Q5) (a) Volume of cylinder $= \pi r^2 h = V \Rightarrow \pi x^2 h = 60 \Rightarrow h = \frac{60}{\pi x^2}$

(b) Surface Area of cylinder $= \underbrace{2\pi r^2}_{\text{Circles}} + \underbrace{2\pi r h}_{\text{Rectangle (curved)}} = 2\pi x^2 + 2\pi x h$

but $h = \frac{60}{\pi x^2} \Rightarrow A = 2\pi x^2 + 2\pi x\left(\frac{60}{\pi x^2}\right) = 2\pi x^2 + \frac{120}{x}$

(c) $A = 2\pi x^2 + 120x^{-1}$

$\frac{dA}{dx} = 4\pi x - 120x^{-2} = 0$ At Minimum

$4\pi x = \frac{120}{x^2} \Rightarrow x^3 = \frac{30}{\pi}, \Rightarrow x = \sqrt[3]{\frac{30}{\pi}}$

$\underline{x = 2.12157\ldots \text{mm}}$

(d) when $x = 2.12 \Rightarrow A = 2\pi x^2 + \frac{120}{x} = 2\pi(2.12)^2 + \frac{120}{2.12} = \underline{84.8\,\text{mm}^2}$

$A = 85\,\text{mm}^2$ to Nearest integer.

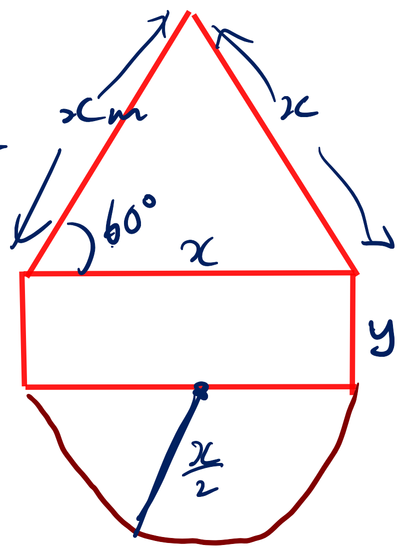
(e) $\frac{d^2A}{dx^2} = 4\pi + 240x^{-3} > 0 \quad \forall x > 0$

for all

OR sub $x = 2.12 \Rightarrow \frac{d^2A}{dx^2} = 4\pi + \frac{240}{(2.12)^3} = 12\pi > 0$ ∴ Minimum turning point.

(8) Total Area = Area of: $\Delta + \square +$ ◡

(a) $A = \frac{1}{2}x^2\sin 60 + xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$



$50 = \frac{1}{2}x^2\frac{\sqrt{3}}{2} + xy + \frac{1}{8}\pi x^2$

$xy = 50 - \frac{1}{4}x^2\sqrt{3} - \frac{1}{8}\pi x^2$

$y = \frac{50}{x} - \frac{2\sqrt{3}x}{8} - \frac{\pi x}{8}$

$y = \frac{50}{x} - \frac{x}{8}(2\sqrt{3} + \pi)$

(b) $P = 2x + 2y + \frac{1}{2}\left(2\pi\left(\frac{x}{2}\right)\right) = 2x + 2y + \frac{\pi}{2}x$

$$P = 2x + \frac{\pi}{2}x + \frac{100}{x} - \frac{x}{4}(2\sqrt{3} + \pi)$$

$$P = \frac{100}{x} + \frac{2\pi x}{4} + \frac{8x}{4} - \frac{\pi x}{4} - \frac{2\sqrt{3}x}{4}$$

$$P = \frac{100}{x} + \frac{\pi x}{4} + \frac{8x}{4} - \frac{2\sqrt{3}x}{4}$$

$$P = \frac{100}{x} + \frac{x}{4}(\pi + 8 - 2\sqrt{3})$$

(c) $\frac{dP}{dx} = -100x^{-2} + \frac{1}{4}(\pi + 8 - 2\sqrt{3}) = 0$ At minimum.

$$\frac{100}{x^2} = \frac{1}{4}(\pi + 8 - 2\sqrt{3})$$

$$x^2 = \frac{400}{\pi + 8 - 2\sqrt{3}}$$

$$x = \sqrt{\frac{400}{\pi + 8 - 2\sqrt{3}}} = \underline{\underline{7.22\,m}}$$

when $x = 7.22 \Rightarrow P = \frac{100}{7.22} + \frac{7.22}{4}(\pi + 8 - 2\sqrt{3})$

$$\underline{P = 27.7\,m}$$

(d) $\frac{d^2P}{dx^2} = 200x^{-3} = \frac{200}{x^3} > 0 \quad \forall x > 0$

$\therefore$ $\underline{P = 27.7\,m \text{ must be a Minimum}}$

$y = e^x \rightarrow \dfrac{dy}{dx} = e^x$

$y = Ae^{kx} \rightarrow \dfrac{dy}{dx} = Ake^{kx}$

$y = a^x \Rightarrow$

$x = \log_a y$

$y = e^x$

$x = \log_e y$

$\underline{x = \ln y}$

(Q11) (a) $m = pe^{-kt}$, $m = 7.5$ when $t = 0$

$\frac{15}{2} = pe^0 \Rightarrow p = \frac{15}{2}$

$m = \frac{15}{2}e^{-kt}$

(b) when $t = 4$ $m = \frac{5}{2}$

$\frac{5}{\cancel{2}} = \frac{15}{\cancel{2}}e^{-4k} \Rightarrow \frac{1}{3} = e^{-4k} \Rightarrow -4k = \ln\left(\frac{1}{3}\right)$

$-4k = \ln 3^{-1}$

$-4k = -\ln 3$

$4k = \ln 3$

$\underline{k = \frac{1}{4}\ln 3}$

$m = \frac{15}{2}e^{-kt}$ where $k = \frac{1}{4}\ln 3$

(c) $\dfrac{dm}{dt} = \frac{15}{2}(-k)e^{-kt} = -\frac{15}{2}ke^{-kt} = -\frac{15}{2}\left(\frac{1}{4}\ln 3\right)e^{-kt} = -\frac{15}{8}(\ln 3)e^{-kt}$

when $\dfrac{dm}{dt} = -\frac{3}{5}\ln 3 \Rightarrow -\frac{15}{8}\cancel{\ln 3}\,e^{-kt} = -\frac{3}{5}\cancel{\ln 3}$

$\Rightarrow e^{-kt} = \dfrac{8}{25} \Rightarrow kt = \ln\left(\frac{25}{8}\right)$

but $k = \frac{1}{4}\ln 3 \Rightarrow \frac{1}{4}\ln 3\, t = \ln\left(\frac{25}{8}\right)$

$\Rightarrow t = 4.15$