

Law of Indices and Fractional Indices – Higher International GCSE (9-1) Exam Questions

Q1.

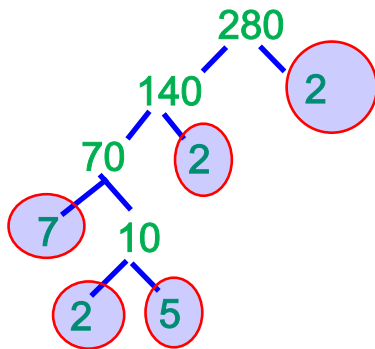
(a) Write $2^3 \times 2^4$ as a single power of 2

$$2^3 \times 2^4 = 2^{3+4} = 2^7$$

.....
(1)

(b) $280 = 2^n \times 5 \times 7$

Find the value of n .



$n = 3$
(2)

(Total for Question is 3 marks)

Q2.

(a) Simplify $e^9 \div e^5$

$$e^9 \div e^5 = e^{9-5} = e^4$$

.....
(1)

(b) Simplify $(y^2)^8$

$$(y^2)^8 = y^{2 \times 8} = y^{16}$$

.....
(1)

(c) Expand and simplify $(x+9)(x-2)$

$$(x+9)(x-2)$$

$$x^2 + 9x - 2x - 18$$

$$x^2 + 7x - 18$$

.....
(2)

(d) Factorise fully $16c^4p^2 + 20cp^3$

$$16c^4p^2 + 20cp^3 = 4cp^2(4c^3 + 5p)$$

.....
(2)

(Total for question = 6 marks)

Q3.(a) Simplify n^0

$$n^0 = 1$$

(1)

(b) Simplify $(3x^2y^5)^3$

$$(3x^2y^5)^3 = 3^3(x^2)^3(y^5)^3 = 27x^6y^{15}$$

(2)

(c) Factorise fully $2e^2 - 18$

(2)

(d) Make r the subject of

$$m = \sqrt{\frac{6a+r}{5r}}$$

$$m = \sqrt{\frac{6a+r}{5r}}$$

$$m^2 = \frac{6a+r}{5r}$$

$$5rm^2 = 6a+r$$

$$5rm^2 - r = 6a$$

$$r(5m^2 - 1) = 6a$$

$$r = \frac{6a}{5m^2 - 1}$$

(4)

(Total for question = 9 marks)**Q4.**(a) Write $3 \times 3 \times 3 \times 3 \times 3$ as a single power of 3

$$3 \times 3 \times 3 \times 3 \times 3 = 3^5$$

(1)

(b) Write $\frac{7^5 \times 7^9}{7^6}$ as a single power of 7

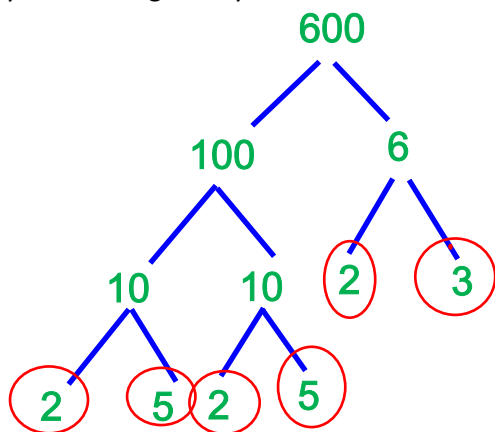
$$\frac{7^5 \times 7^9}{7^6} = \frac{7^{14}}{7^6} = 7^8$$

(2)

(Total for Question is 3 marks)

Q5.

- (a) Express 600 as a product of powers of its prime factors.
Show your working clearly.



$$600 = 2^3 \times 3 \times 5^2$$

.....
(3)

- (b) Simplify

$$\frac{5^{12}}{5^2 \times 5}$$

Give your answer as a power of 5

$$\frac{5^{12}}{5^2 \times 5} = \frac{5^{12}}{5^3} = 5^{12-3} = 5^9$$

.....
(2)

(Total for question = 5 marks)

Q6.

- (a) Write $2^3 \times 2^6$ as a single power of 2

$$2^3 \times 2^6 = 2^9$$

.....
(1)

- (b) Write $\frac{3^9}{3^4}$ as a single power of 3

$$\frac{3^9}{3^4} = 3^5$$

.....
(1)

- (c)

$$\frac{5^n}{5^4 \times 5^6} = 5^3$$

Find the value of n .

$$\frac{5^n}{5^4 \times 5^6} = 5^3$$

$$\frac{5^n}{5^{10}} = 5^3$$

$$5^n = 5^{10} \times 5^3$$

$$5^n = 5^{13}$$

$$n = 13$$

$n =$
(2)

(Total for question = 4 marks)

Q7.

(a) Simplify, leaving your answers in index form,

(i) $6^5 \times 6^2 \times 6$

$$6^5 \times 6^2 \times 6 = 6^{5+2+1} = 6^8$$

(ii) $(9^7)^2$

$$9^{7 \times 2} = 9^{14}$$

.....

.....

(2)

(b)

$$\frac{5^n \times 5^3}{5^6} = 5^4$$

Find the value of n .

$$\frac{5^n \times 5^3}{5^6} = 5^4$$

$n =$

(2)

$$5^{n+3} = 5^{10}$$

$$n + 3 = 10$$

$$n = 7$$

(Total for question = 4 marks)

Q8.

(a) Simplify $e^8 \times e^7$

$$e^8 \times e^7 = e^{8+7} = e^{15}$$

.....

(1)

(b) Simplify fully

$$\frac{12g^{10}}{3g^2}$$

$$\frac{12g^{10}}{3g^2} = 4g^8$$

.....


(2)

(c) Write down the value of m^0

$$m^0 = 1$$

.....

(1)

 (d) Simplify fully $(27x^6)^{\frac{2}{3}}$

$$(27x^6)^{\frac{2}{3}} = 27^{\frac{2}{3}} (x^6)^{\frac{2}{3}} = 3^2 (x^4) = 9x^4$$

.....

(2)

(Total for question = 6 marks)

Q9.

(a) Simplify $(16x^4y^2)^{\frac{1}{2}}$

$$(16x^4y^2)^{\frac{1}{2}} = 16^{\frac{1}{2}}(x^4)^{\frac{1}{2}}(y^2)^{\frac{1}{2}} = 4x^2y$$

.....
(2)

(b) Simplify fully

$$\frac{2x^2 - 8}{4x^2 - 8x}$$

$$\frac{2(x^2 - 4)}{4x(x - 2)} = \frac{(x + 2)(x - 2)}{2x(x - 2)} = \frac{x + 2}{2x}$$

.....
(3)

(Total for Question is 5 marks)

Q10.

(a) Show that $(6 + 2\sqrt{12})^2 = 12(7 + 4\sqrt{3})$

Show each stage of your working.

$$(6 + 2\sqrt{12})^2 = 12(7 + 4\sqrt{3})$$

$$(6 + 2\sqrt{12})^2 = (6 + 2\sqrt{4}\sqrt{3})^2 = (6 + 4\sqrt{3})^2$$

$$(6 + 4\sqrt{3})^2 = (6 + 4\sqrt{3})(6 + 4\sqrt{3}) = 36 + 24\sqrt{3} + 24\sqrt{3} + 48 = 84 + 48\sqrt{3} = 12(7 + 4\sqrt{3})$$

.....
(3)

(b) Simplify fully $\left(\frac{27a^{12}}{t^{15}}\right)^{-\frac{2}{3}}$

$$\left(\frac{27a^{12}}{t^{15}}\right)^{-\frac{2}{3}} = \left(\frac{t^{15}}{27a^{12}}\right)^{\frac{2}{3}} = \frac{(t^{15})^{\frac{2}{3}}}{(27)^{\frac{2}{3}}(a^{12})^{\frac{2}{3}}} = \frac{t^{10}}{9a^8}$$

.....
(3)

(Total for question = 6 marks)

Q11.

(a) Simplify $(16y^8)^{\frac{3}{4}}$

$$(16y^8)^{\frac{3}{4}} = 16^{\frac{3}{4}}(y^8)^{\frac{3}{4}} = \left(\sqrt[4]{16}\right)^3 y^{\frac{3 \times 8}{4}} = 2^3 y^6 = 8y^6$$

.....
(2)

(b) Given that $2^p \times 8^q = 2^n$

express n in terms of p and q .

$$2^p \times 8^q = 2^n$$

$$2^p \times (2^3)^q = 2^n$$

$$2^p \times 2^{3q} = 2^n$$

$$2^{p+3q} = 2^n$$

$$n = p + 3q$$

$n =$
(2)

$$\frac{8}{2^7} = 2^n \quad \text{(Total for question is 4 marks)}$$

Q12.

$$\frac{8}{2^7} = 2^n$$

$$\frac{2^3}{2^7} = 2^n$$

(a) Find the value of n .

$$2^{-4} = 2^n$$

$$n = -4$$

$n =$
(2)

$$(13^{-6})^4 \times 13^5 = 13^k$$

(b) Find the value of k .

$$(13^{-6})^4 \times 13^5 = 13^k$$

$$13^{-24} \times 13^5 = 13^k$$

$$13^{-19} = 13^k$$

$$k = -19$$

$k =$
(2)

(Total for question = 4 marks)

Q13.

(a) $A = 2^2 \times 3 \times 5^2$
 $B = 2^3 \times 5$

$$2 \times 2 \times 5 = 20$$

$$HCF = 2^2 \times 5$$

(i) Find the Highest Common Factor (HCF) of A and B.

.....

(ii) Find the Lowest Common Multiple (LCM) of A and B.

$$LCM = 2^3 \times 3 \times 5^2$$

.....

(3)

(b)

$$\frac{8^2 \times 8^3}{8^4} = 2^n$$

Find the value of n .

$$\frac{8^2 \times 8^3}{8^4} = 2^n$$

$$\frac{8^5}{8^4} = 2^n$$

$$8 = 2^n$$

$$2^3 = 2^n$$

$$n = 3$$

$n =$

(2)

(Total for question = 5 marks)

Q14.

(a) Write $\frac{1}{32}$ as a power of 2

$$2^5 = 32 \Rightarrow 2^{-5} = \frac{1}{32}$$

.....

(2)

(b) Show that $(4 + \sqrt{12})(5 - \sqrt{3}) = 14 + 6\sqrt{3}$

Show each stage of your working clearly.

$$(4 + \sqrt{12})(5 - \sqrt{3}) = 14 + 6\sqrt{3}$$

$$(4 + \sqrt{4} \sqrt{3})(5 - \sqrt{3}) = (4 + 2\sqrt{3})(5 - \sqrt{3})$$

$$20 + 10\sqrt{3} - 4\sqrt{3} - 6 = 14 + 6\sqrt{3}$$

(3)

(Total for question = 5 marks)

Q15.

$$\frac{1}{5^3} = 5^p \quad 1 = 5^q \quad \sqrt{5^3} = 5^r$$

(a) Write down the value of

(i) p $\frac{1}{5^3} = 5^p$
 $5^{-3} = 5^p \Rightarrow p = -3$ $p = \dots\dots\dots$

(ii) q $1 = 5^q$
 $5^0 = 5^q \Rightarrow q = 0$ $q = \dots\dots\dots$

(iii) r $\sqrt{5^3} = 5^r$
 $5^{\frac{3}{2}} = 5^r \Rightarrow r = \frac{3}{2}$ $r = \dots\dots\dots$ (3)

(b) Show that

$$\frac{14}{\sqrt{245}} = \frac{2\sqrt{5}}{5}$$

You must write down each stage of your working.

$$\frac{14}{\sqrt{245}} = \frac{14}{\sqrt{49} \sqrt{5}} = \frac{14}{7\sqrt{5}} = \frac{2}{\sqrt{5}}$$

$$\frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

(2)

$(e - 2\sqrt{3})^2 = f - 20\sqrt{3}$ where e and f are integers.

(c) Find the value of e and the value of f

$$(e - 2\sqrt{3})^2 = f - 20\sqrt{3}$$

$$e^2 - 4e\sqrt{3} + 12 = f - 20\sqrt{3}$$

compare irrational parts

$$4e = 20 \Rightarrow e = 5$$

compare rational parts

$$\Rightarrow e^2 + 12 = f$$

$$\text{but } e = 5$$

$$\therefore 5^2 + 12 = f$$

$$25 + 12 = f$$

$$f = 37$$

$$e = \dots\dots\dots$$

$$f = \dots\dots\dots$$

(3)

(Total for question = 8 marks)

Q16.

(a) Simplify $(\sqrt{x})^8$

$$(\sqrt{x})^8 = (x^{\frac{1}{2}})^8 = x^4$$

.....
(1)

(b) Solve

$$\frac{6 + 4y}{3} = 5 - 2y$$

Show clear algebraic working.

$$6 + 4y = 3(5 - 2y)$$

$$4y = 15 - 6y - 6$$

$$4y + 6y = 9$$

$$10y = 9$$

$$y = \frac{9}{10}$$

$y =$
(4)

(c) Make g the subject of $g - 1 = gh + 3h$

$$g - 1 = gh + 3h$$

$$g = gh + 3h + 1$$

$$g - gh = 3h + 1$$

$$g(1 - h) = 3h + 1$$

$$g = \frac{3h + 1}{1 - h}$$

.....
(3)

(Total for question = 8 marks)

Q17.

(a) Factorise $2x^2 - 7x + 6$

$$(2x - 3)(x - 2)$$

.....
(2)

(b) Solve

$$\frac{4m + 9}{3} = 7 - 2m$$

Show clear algebraic working.

$$4m + 9 = 3(7 - 2m)$$

$$4m = 21 - 6m - 9$$

$$10m = 12$$

$$m = \frac{12}{10} = 1.2$$

$m =$
(4)

(c) Write $\frac{\sqrt[4]{y}}{y}$ in the form y^b where b is a fraction.

$$\frac{\sqrt[4]{y}}{y} = y^{\frac{1}{4} - 1} = y^{-\frac{3}{4}}$$

.....
(2)

$$3 \times 4^{2k+8} = 24$$

$$4^{2k+8} = 8$$

$$(2^2)^{2k+8} = 2^3$$

$$2^{4k+16} = 2^3$$

$$4k + 16 = 3$$

$$k = -\frac{13}{4}$$

$k =$

(Total for question = 8 marks)

(Total for question = 4 marks)

Q18. Solve $3 \times 4^{2k+8} = 24$

Show your working clearly.

Q19. Given that $\left(2^{\frac{1}{2}}\right)^n = \frac{2^x}{8^y}$

express n in terms of x and y .

$$\left(2^{\frac{1}{2}}\right)^n = \frac{2^x}{(2^3)^y}$$

$$2^{\frac{n}{2}} = \frac{2^x}{2^{3y}}$$

$$2^n = 2^{x-3y}$$

$$n = x - 3y$$

.....

(Total for Question is 3 marks)

Q20.

$$g = 2^3 \times 3 \times 7^2 \quad h = 2 \times 3 \times 7^3$$

(a) Express gh as a product of powers of its prime factors.

Simplify your answer.

$$gh = 2^3 \times 3 \times 7^2 \times 2 \times 3 \times 7^3 = 2^{3+1} \times 3^{1+1} \times 7^{2+3} = 2^4 \times 3^2 \times 7^5$$

.....
(2)

$$\frac{g}{h} = 2^a \times 3^b \times 7^c$$

(b) Find the value of a , the value of b and the value of c .

$$g = 2^3 \times 3 \times 7^2 \quad h = 2 \times 3 \times 7^3$$

$$\frac{g}{h} = \frac{2^3 \times 3 \times 7^2}{2 \times 3 \times 7^3} = 2^2 \times 3^0 \times 7^{-1}$$

compare with $2^a \times 3^b \times 7^c$

$$a = 2 \quad b = 0 \quad c = -1$$

$a =$

$b =$

$c =$

(2)

(c) Show that $(7 - 2\sqrt{5})(7 + 2\sqrt{5}) = 29$

Show your working clearly.

$$(7 - 2\sqrt{5})(7 + 2\sqrt{5})$$

$$49 - 14\sqrt{5} + 14\sqrt{5} - 20 = 29$$

(2)

$$\frac{1}{\sqrt[3]{9^4}} = 3^n$$

(d) Work out the exact value of n .

$$\frac{1}{\sqrt[3]{9^4}} = 3^n$$

$$\frac{1}{\sqrt[3]{(3^2)^4}} = \frac{1}{\sqrt[3]{3^8}} = \frac{1}{3^{\frac{8}{3}}} = 3^{-\frac{8}{3}}$$

$$n = -\frac{8}{3}$$

.....
(3)

(Total for question = 9 marks)

Q21.

$$(3 + \sqrt{c})(2\sqrt{c} - 3) = 1 + k\sqrt{c}$$

where c and k are prime numbers.

(a) Find the value of c and the value of k .

$$(3 + \sqrt{c})(2\sqrt{c} - 3) = 1 + k\sqrt{c}$$

$$6\sqrt{c} - 9 + 2c - 3\sqrt{c} = 1 + k\sqrt{c}$$

$$3\sqrt{c} - 9 + 2c = 1 + k\sqrt{c}$$

$$\text{equate rational terms; } -9 + 2c = 1 \Rightarrow c = 5$$

$$\text{equate irrational terms; } 3\sqrt{c} = k\sqrt{c} \Rightarrow k = 3$$

$$c = \dots\dots\dots 5 \dots\dots\dots k = \dots\dots\dots 3 \dots\dots\dots$$

(3)

$$p^m = \frac{1}{p \times \sqrt[3]{p^2}}$$

(b) Find the value of m .

$$p^m = \frac{1}{p \times \sqrt[3]{p^2}} = \frac{1}{p \times p^{\frac{2}{3}}} = \frac{1}{p^{\frac{2}{3} + 1}} = \frac{1}{p^{\frac{5}{3}}} = p^{-\frac{5}{3}}$$

$$m = -\frac{5}{3}$$

$$m = \dots\dots\dots$$

(3)

(Total for question = 6 marks)

Q22.

(a) Simplify $(4h^{\frac{2}{3}})^3$

$$\left(4h^{\frac{2}{3}}\right)^3 = 4^3\left(h^{\frac{2}{3}}\right)^3 = 64h^2$$

.....
(2)

(b) Work out the value of k .

$$\frac{a\sqrt{a}}{\sqrt[3]{a^2}} = a^k$$

$$\frac{a\sqrt{a}}{\sqrt[3]{a^2}} = a^k$$

$$\frac{a \times a^{\frac{1}{2}}}{a^{\frac{2}{3}}} = a^k$$

$$a^{\frac{9}{6} - \frac{4}{6}} = a^k$$

$$a^{\frac{5}{6}} = a^k$$

$$k = \dots\dots\dots$$

(3)

$$\frac{a^{\frac{3}{2}}}{a^{\frac{2}{3}}} = a^k$$

$$k = \frac{5}{6}$$

(Total for question = 5 marks)

Q23.

Solve the equation

$$\frac{2^{(n^2)}}{2^n \times 2^6} = 1$$

Show clear algebraic working.

$$\frac{2^{(n^2)}}{2^n \times 2^6} = 1$$

$$2^{n^2} = 2^{n+6}$$

$$n^2 - n - 6 = 0$$

$$(n - 3)(n + 2) = 0$$

$$n = 3 \text{ or } n = -2$$

.....
(Total for Question is 3 marks)

Q24.

$m = 8 \times 10^{9n}$ where n is an integer.

Express $m^{-\frac{1}{3}}$ in standard form.

Give your answer, in terms of n , as simply as possible.

$$m = 8 \times 10^{9n}$$

$$m^{-\frac{1}{3}} = 8^{-\frac{1}{3}} \times (10^{9n})^{-\frac{1}{3}}$$

$$m^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{8}} \times (10^{-3n})$$

$$m^{-\frac{1}{3}} = \frac{1}{2} \times 10^{-3n}$$

$$m^{-\frac{1}{3}} = 0.5 \times 10^{-3n}$$

$$m^{-\frac{1}{3}} = 5 \times 10^{-3n-1}$$

standard form

.....
(Total for question = 3 marks)