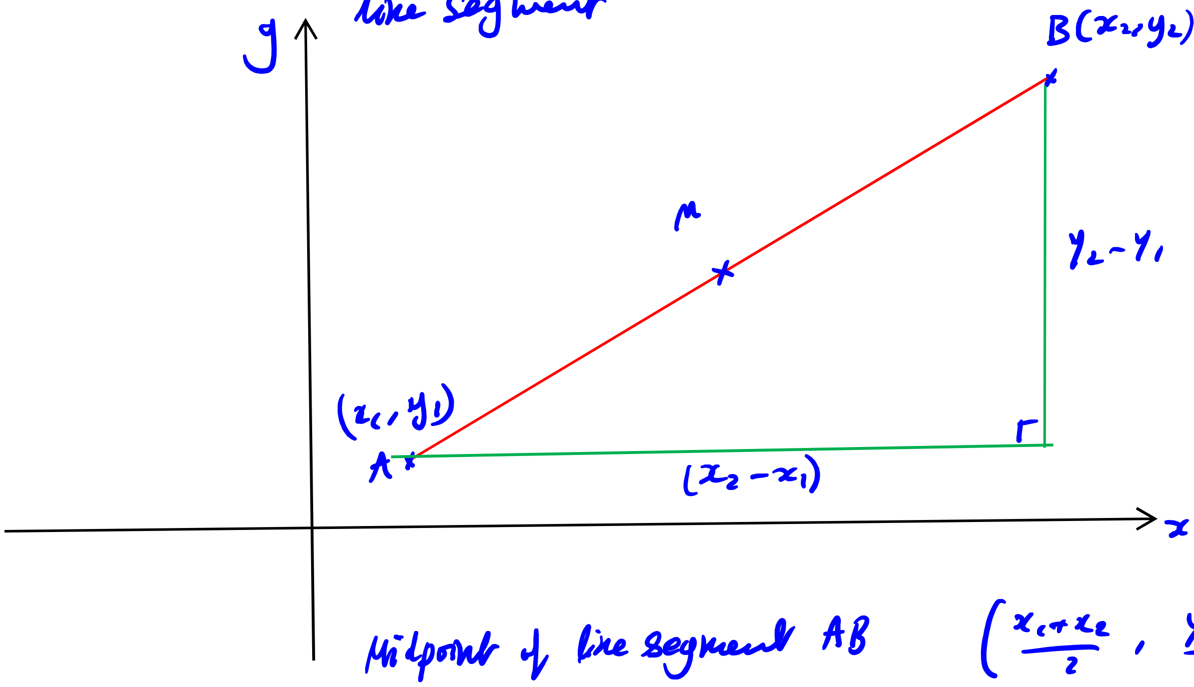
Coordinate Geometry

15/06/20

Line segment



Midpoint of line segment AB $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Gradient of AB $= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

length of a line segment $l^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Equation of a line can be written as $y = mx + c$ or $ax + by + c = 0$
where a, b and c are integers.

given two points we can find $m = \frac{y_2 - y_1}{x_2 - x_1}$

then we can use $y - y_1 = m(x - x_1)$ to find the eq^n of the line.

$y = mx + c$ $\quad$ $ax + by + c = 0$

if we know the gradient of line 1 is m_1
then if line 2 is parallel to l_1 then $m_1 = m_2$
if line 2 is perpendicular to l_1 then $m_2 = -\frac{1}{m_1}$ {Negative Reciprocal}.

WSA

Q4) (a) $m = \frac{1}{2}$ $(1,6)$

$y - y_1 = m(x - x_1)$

$y - 6 = \frac{1}{2}(x - 1)$

Q5(e) $y = mx + c$

$m = \frac{1}{3}$ $(-3, 1)$

$y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{3}(x + 3)$

$y = \frac{1}{3}x + 1 + 1$

$y = \frac{1}{3}x + 2$

Q6) $ax + by + c = 0$

(d) $m = \frac{2}{5}$ $P(-3, 5)$

$y - y_1 = m(x - x_1)$

$y - 5 = \frac{2}{5}(x + 3)$

$5y - 25 = 2x + 6$

$2x - 5y + 31 = 0$

Q7) (d) $(-\frac{1}{2}, -2)$ $(2, 8)$

$m = \frac{\Delta y}{\Delta x} = \frac{8 - (-2)}{2 - (-\frac{1}{2})} = \frac{10 \times 2}{5} = 4$

$m = 4$ $(2, 8)$

$y - y_1 = m(x - x_1)$

$y - 8 = 4(x - 2)$

$y - 8 = 4x - 8$

$y = 4x$

8(d) $(-4, -1)$ $(8, -17)$

$m = \frac{-17 - (-1)}{8 - (-4)} = \frac{-16}{12} = -\frac{4}{3}$

$(8, -17)$ $m = -\frac{4}{3}$

$y - y_1 = m(x - x_1)$

$y + 17 = -\frac{4}{3}$

⋮

$4x + 3y + 19 = 0$

18 (f) $(-1, -2)$ $(4, -5)$

$M\left(\frac{-1+4}{2}, \frac{-2-5}{2}\right)$

$M\left(\frac{3}{2}, \frac{-7}{2}\right)$

(14)(e) $l^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$(3, 12)$ $(1, 7)$

$l^2 = (3-1)^2 + (12-7)^2$

$l^2 = 2^2 + 5^2$

$l^2 = 4 + 25$

$l = \sqrt{29}$

(19) l_1; $P(-2, 1)$ $Q(4, -1)$

(a) $m = \frac{\Delta y}{\Delta x} = \frac{1 - (-1)}{-2 - 4} = \frac{2}{-6} = -\frac{1}{3}$

$y - y_1 = m(x - x_1)$

$y + 1 = -\frac{1}{3}(x - 4)$

$3y + 3 = -x + 4$

$x + 3y - 1 = 0$

(b) midpoint of PQ $m = \left(\frac{-2+4}{2}, \frac{1-1}{2}\right)$

$M(1, 0)$

$M(1, 0)$ $R(2, 4)$

Grad of MR $= \frac{4-0}{2-1} = 4$

$y - y_1 = m(x - x_1)$

$y = 4(x - 1)$ $\Rightarrow$ $y = 4x - 4$

WSB

1(d) $x+2y-3=0$

$2y=-x+3$

$y=-\frac{1}{2}x+3$

Gradient is $-\frac{1}{2}$

Gradient of perpendicular line is $\underline{\underline{2}}$

(Q3)(b) $3x+4y=1$

$4y=-3x+1$

$y=-\frac{3}{4}x+\frac{1}{4}$

Gradient $=-\frac{3}{4}$

Grad of $\perp = \frac{4}{3}$ $\quad(2,5)$

$y-y_1=m(x-x_1)$

$y-5=\frac{4}{3}(x-2)$

$3y-15=4x-8$

$\underline{4x-3y+7=0}$

Q4) (b) $(2,7)$ $\quad(4,1)$

$M\left(\frac{2+4}{2},\frac{7+1}{2}\right)$

$M(3,4)$

Gradient $=\frac{\Delta y}{\Delta x}=\frac{7-1}{2-4}=\frac{6}{-2}=-3$

Grad of $\perp = \frac{1}{3}$

$m=\frac{1}{3}$ $\quad M(3,4)$

$y-y_1=m(x-x_1)$

$y-4=\frac{1}{3}(x-3)$

$3y-12=x-3$

$\underline{x-3y+9=0}$

WSB

17/06/20

Q6

$2x-3y+5=0$ $\quad\perp$ to $\quad 3x+ky-1=0$

$3y=2x+5$

$y=\frac{2}{3}x+\frac{5}{3}$

$ky=1-3x$

$y=-\frac{3}{k}x+\frac{1}{k}$.

lines are perpendicular $\therefore$ $\frac{2}{3}\times-\frac{3}{k}=-1 \Rightarrow \underline{\underline{k=2}}$

$\Rightarrow \underline{\underline{m_1\times m_2=-1}}$

9 .... l_1 ; $3x+y+2=0$ —— ①

l_2 ; $x-3y+14=0$ $\quad\times 3$

$3x-9y+42=0$ —— ②

① − ②

$10y=40$

$y=4 \Rightarrow x-3(4)+14=0$

$\underline{x=-2}$

$(-2,4)$

distance from Origin $l^2 = \sqrt{(-2)^2 + 4^2} = \sqrt{4+16} = \sqrt{20} = \underline{2\sqrt{5}}$

WSC 22/06/20

Q4) l; $3x + y - 9 = 0$ —— (1)

m; $2x + 3y - 12 = 0$ —— (2)

(a) Meet at

(1) $\times 3$ $\quad 9x + 3y - 27 = 0$

$2x + 3y - 12 = 0$

subtract $\quad 7x - 15 = 0$

$\underline{x = \frac{15}{7}}$

$\Rightarrow 2\left(\frac{15}{7}\right) + 3y - 12 = 0$

$\frac{30}{7} + 3y - 12 = 0$

$3y = 12 - \frac{30}{7}$

$y = \frac{84 - 30}{21}$

$y = \frac{54}{21} = \frac{18}{7}$

$\underline{\left(\frac{15}{7}, \frac{18}{7}\right)}$

(b) l crosses x-axis at $y=0$

$9x = 27 \Rightarrow \underline{x = 3}$ $\quad (3,0)$

l crosses y-axis at $x = 0$

$3y = 27 \Rightarrow y = 9$ $\quad (0,9)$

m crosses y-axis at $x = 0$

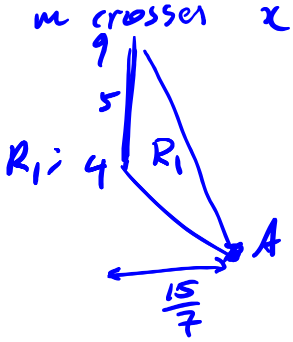
$3y = 12$

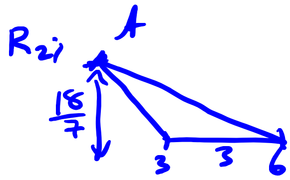
$y = 4$ $\quad (0,4)$

m crosses x-axis at $y = 0$

$2x = 12$

$x = 6$ $\quad (6,0)$





Area of R_1

$\frac{1}{2} \times 5 \times \frac{15}{7} = \frac{75}{14}$

Area of R_2

$\frac{1}{2} \times 3 \times \frac{18}{7} = \frac{54}{14}$

Ratio of areas.

$R_1 : R_2$

$\frac{75}{14} : \frac{54}{14}$

$75 : 54$

$\underline{25 : 18}$

Q7) $A(-8,1)$ $\quad B(-4,-5)$

(a) Grad of $AB = \frac{\Delta y}{\Delta x} = \frac{-5-1}{-4-(-8)} = \frac{-6}{4} = -\frac{3}{2}$

$m = -\frac{3}{2}$ $\quad A(-8,1)$

$y - 1 = -\frac{3}{2}(x+8)$

$2y - 2 = -3x - 24$

$\underline{3x + 2y + 22 = 0}$

(b) Midpoint of $AB = \left(\frac{-8+-4}{2}, \frac{1+-5}{2}\right) = (-6,-2)$

$l^2 = (-6)^2 + (-2)^2 = 36 + 4 = 40$

$l = \sqrt{40} = \underline{2\sqrt{10}}$ $\quad \therefore \underline{k = 2}$.

Q10) $A(1, 2\sqrt{3})$ $\quad B(\sqrt{3}, 6)$

(a) Gradient of $l = \frac{\Delta y}{\Delta x} = \frac{6 - 2\sqrt{3}}{\sqrt{3} - 1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{6\sqrt{3} - 6 + 6 - 2\sqrt{3}}{3 - \cancel{\sqrt{3}} + \cancel{\sqrt{3}} - 1} = \frac{4\sqrt{3}}{2} = \underline{2\sqrt{3}}$

(b) $m = 2\sqrt{3}$ $A(1, 2\sqrt{3})$

eqn of line l

$$y - y_1 = m(x - x_1)$$
$$y - 2\sqrt{3} = 2\sqrt{3}(x-1)$$
$$y - \cancel{2\sqrt{3}} = 2x\sqrt{3} - \cancel{2\sqrt{3}}$$
$$\underline{y = 2x\sqrt{3}}$$

passes through $(0,0)$ origin.

(c) perpendicular to l

$\Rightarrow m = -\frac{1}{2\sqrt{3}}$ $A(1, 2\sqrt{3})$

$$y - y_1 = m(x - x_1)$$
$$y - 2\sqrt{3} = -\frac{1}{2\sqrt{3}}(x-1)$$
$$2y\sqrt{3} - 12 = -x + 1$$
$$\underline{2y\sqrt{3} + x - 13 = 0}$$

As required

WSD

Q5) $2x - y + 4 = 0$ $(-1, -3)$

(a) $y = 2x + 4$

$m = 2$

$$y - y_1 = m(x - x_1)$$
$$\underline{y + 3 = 2(x+1)}$$

(b) $6x + 5y - 2 = 0$

$$5y = -6x + 2$$
$$y = -\frac{6}{5}x + \frac{2}{5}.$$

$m = -\frac{6}{5}$

Grad of perpendicular

$m = \frac{5}{6}$ $(4, 4)$

$$y - y_1 = m(x - x_1)$$
$$y - 4 = \frac{5}{6}(x - 4)$$
$$6y - 24 = 5x - 20$$
$$\underline{5x - 6y + 4 = 0}$$

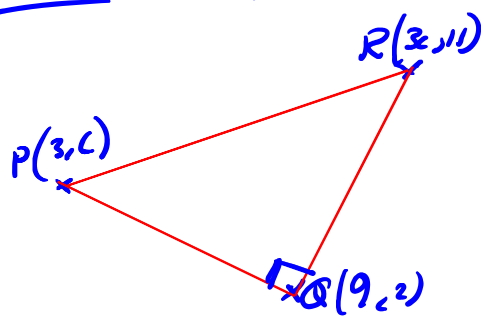
(c) l and m intersect when

$y = 2x - 1$ sub into m

$$5x - 6y + 4 = 0$$
$$5x - 6(2x-1) + 4 = 0$$
$$5x - 12x + 6 + 4 = 0$$
$$-7x + 10 = 0$$
$$\Rightarrow \underline{x = \frac{10}{7}}$$

$$\therefore y = 2x - 1$$
$$y = 2\left(\frac{10}{7}\right) - 1$$
$$y = \frac{20 - 7}{7} = \frac{13}{7}$$

$\underline{\left(\frac{10}{7}, \frac{13}{7}\right)}$, $\underline{\left(1\frac{3}{7}, 1\frac{6}{7}\right)}$



(9) $P(3, c)$ $Q(9, 2)$ $R(3c, 11)$

Grad of $PQ = \frac{\Delta y}{\Delta x} = \frac{c-2}{3-9} = \frac{c-2}{-6} = \frac{2-c}{6}$

Grad of $RQ = \frac{\Delta y}{\Delta x} = \frac{11-2}{3c-9} = \frac{9}{3c-9} = \frac{3}{c-3}$

$$m_1 \times m_2 = -1 \Rightarrow \frac{2-c}{6} \times \frac{3}{c-3} = -1$$
$$3(2-c) = -6(c-3)$$
$$6 - 3c = -6c + 18.$$
$$3c = 12$$
$$\underline{c = 4}$$

(b) $\underline{P(3,4) \quad Q(9,2) \quad R(12,11)}$

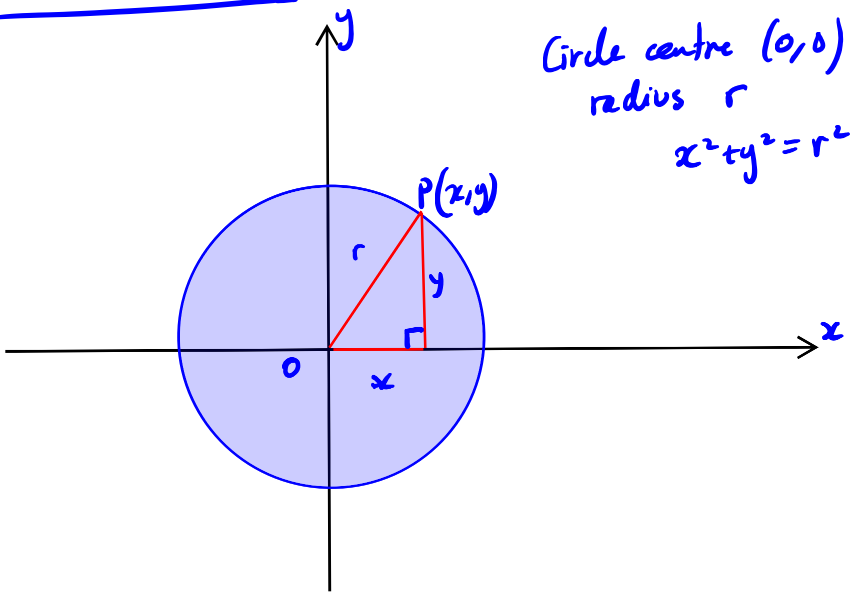
length of PQ.

$$l^2 = (9-3)^2 + (4-2)^2$$
$$l^2 = 36 + 4$$
$$l^2 = 40$$
$$\underline{l = 2\sqrt{10}} \Rightarrow \underline{k = 2}$$

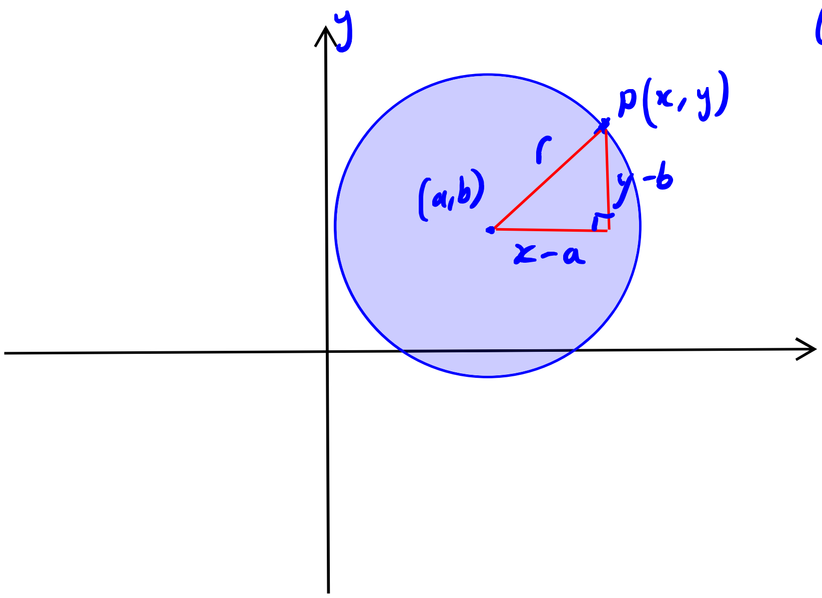
length of QR $\Rightarrow$ $l^2 = (12-9)^2 + (11-2)^2$
$l^2 = 3^2 + 9^2$
$l^2 = 9 + 81$
$l^2 = 90$
$l = 3\sqrt{10}$

Area of $\Delta PQR = \frac{1}{2} bh$
$= \frac{1}{2} \times 3\sqrt{10} \times 2\sqrt{10}$
$= 3 \times 10 = 30 \text{ units}^2$

C2 Solomon — Further Coordinate geometry — 24/06/20



Circle centre (0,0)
radius r
$x^2 + y^2 = r^2$ – Apply pythagoras' theorem.



Circle of radius r, centre (a,b)
apply pythagoras theorem
$(x-a)^2 + (y-b)^2 = r^2$
eqn of a circle (length of a line segment)

WSA

1(e) Centre $\left(-\frac{1}{2}, \frac{1}{2}\right)$ radius $\frac{1}{2}$
$(x-a)^2 + (y-b)^2 = r^2$
$\left(x+\frac{1}{2}\right)^2 + \left(y-\frac{1}{2}\right)^2 = \frac{1}{4}$

$(x-a)^2 + (y-b)^2 = r^2$
2(e) $(x+2)^2 + (y+5)^2 = 32$
Centre $(-2, -5)$ radius $\sqrt{32} = 4\sqrt{2}$

③ (b) $x^2+y^2-2x-10y-23=0$

$x^2-2x+y^2-10y-23=0$

$(x-1)^2-1+(y-5)^2-25-23=0$

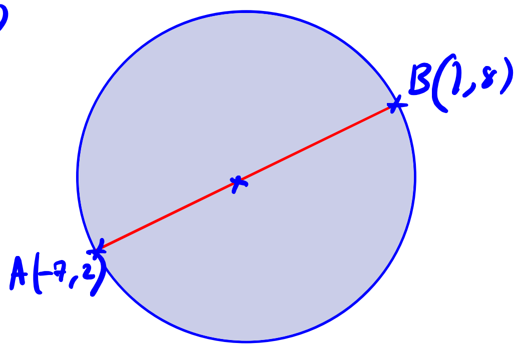
$(x-1)^2+(y-5)^2=49$

Circle centre $(1,5)$ radius 7.

(d) $x^2+y^2-2x+16y=35$

$(1,-8)$ radius 10

5(b)

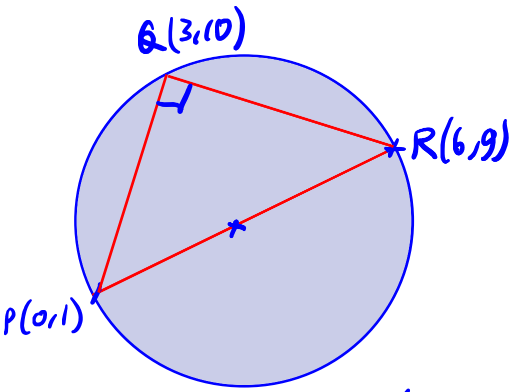


Centre lies at the midpoint of the diameter AB.

midpoint $\left(\frac{-7+1}{2}, \frac{8+2}{2}\right)$

$(-3,5)$

⑥



Grad of PQ $= \frac{10-1}{3-0} = \frac{9}{3} = 3$

Grad of QR $= \frac{9-10}{6-3} = \frac{-1}{3}$

perpendicular if $m_1 \times m_2 = -1$

$-\frac{1}{3} \times 3 = -1$

$\therefore$ PQ is $\perp$ to QR $\therefore$ $\angle PQR = 90°$

Use circle theorem that angle in a semi circle is a right angle

$\therefore$ PR is a diameter.

Midpoint of PR $= \left(\frac{0+6}{2}, \frac{1+9}{2}\right) = (3,5)$.

Eqn of circle $(x-a)^2+(y-b)^2=r^2$

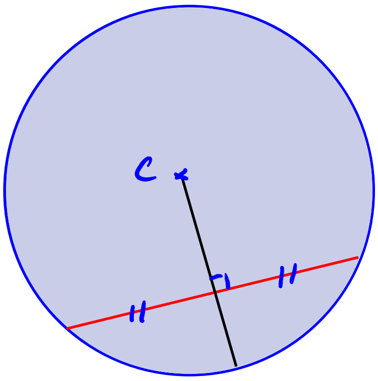
$(x-3)^2+(y-5)^2=r^2$

P(0,1) $(0-3)^2+(1-5)^2=r^2$

$9+16=r^2 \Rightarrow r^2=25$ $r=5$

$(x-3)^2+(y-5)^2=25$ equation of a circle

Centre $(3,5)$ radius 5



the perpendicular bisector of any chord passes through the centre of the circle.

(11) $P(-2,-2)$ $Q(2,-4)$ $R(7,1)$

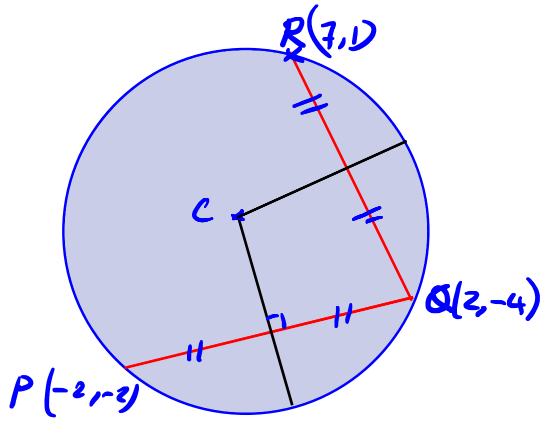
(a) perpendicular bisector of PQ

$$\text{Grad of } PQ = \frac{-4-(-2)}{2-(-2)} = \frac{-4+2}{4} = \frac{-2}{4} = -\frac{1}{2}$$

$$\text{grad of } \perp = 2.$$

$$\text{midpoint of } PQ = \left(\frac{2+-2}{2}, \frac{-2-4}{2}\right) = (0,-3)$$

$$\underline{y = 2x - 3} \quad (1)$$



(b) Eq^n of perpendicular bisector of RQ

$$\text{Grad of } RQ = \frac{1-(-4)}{7-2} = \frac{5}{5} = 1$$

$$\text{grad of } \perp = -1$$

$$\text{Midpoint of } RQ = \left(\frac{7+2}{2}, \frac{-4+1}{2}\right) = \left(\frac{9}{2}, -\frac{3}{2}\right)$$

eq^n

$$y - y_1 = m(x - x_1)$$

$$y + \frac{3}{2} = -\left(x - \frac{9}{2}\right) \quad (2)$$

the centre of the circle lies a the point of intersection of the two perpendicular bisectors.

Solve (1) and (2) simultaneously

$$2x - 3 + \frac{3}{2} = -x + \frac{9}{2}$$

$$4x - 6 + 3 = -2x + 9$$

$$6x = 12$$

$$\underline{x = 2}$$

when $x = 2$

$$\Rightarrow y = 2(2) - 3 = 1$$

Centre of circle $\underline{(2,1)}$

(c) eqn of circle $(x-a)^2+(y-b)^2=r^2$

$(x-2)^2+(y-1)^2=r^2$

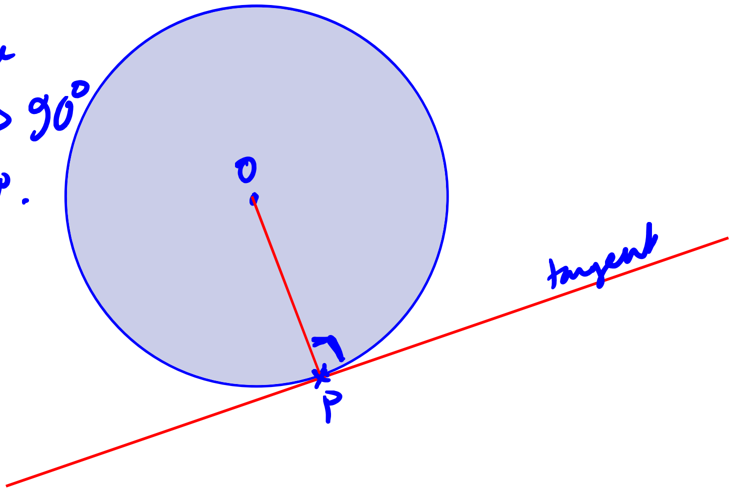
$R(7,1)$ $\quad (7-2)^2+(1-1)^2=r^2$

$25 = r^2 \Rightarrow \underline{r=5}$

$$\underline{(x-2)^2+(y-1)^2=5^2}$$

The angle between a tangent and radius is 90°

OP is the Normal to the tangent at P.



(14) $x^2+y^2-6x-10y+16=0$ $\quad A(6,2)$

(a) Centre of C

$(x-3)^2-9+(y-5)^2-25+16=0$

$(x-3)^2+(y-5)^2=18$

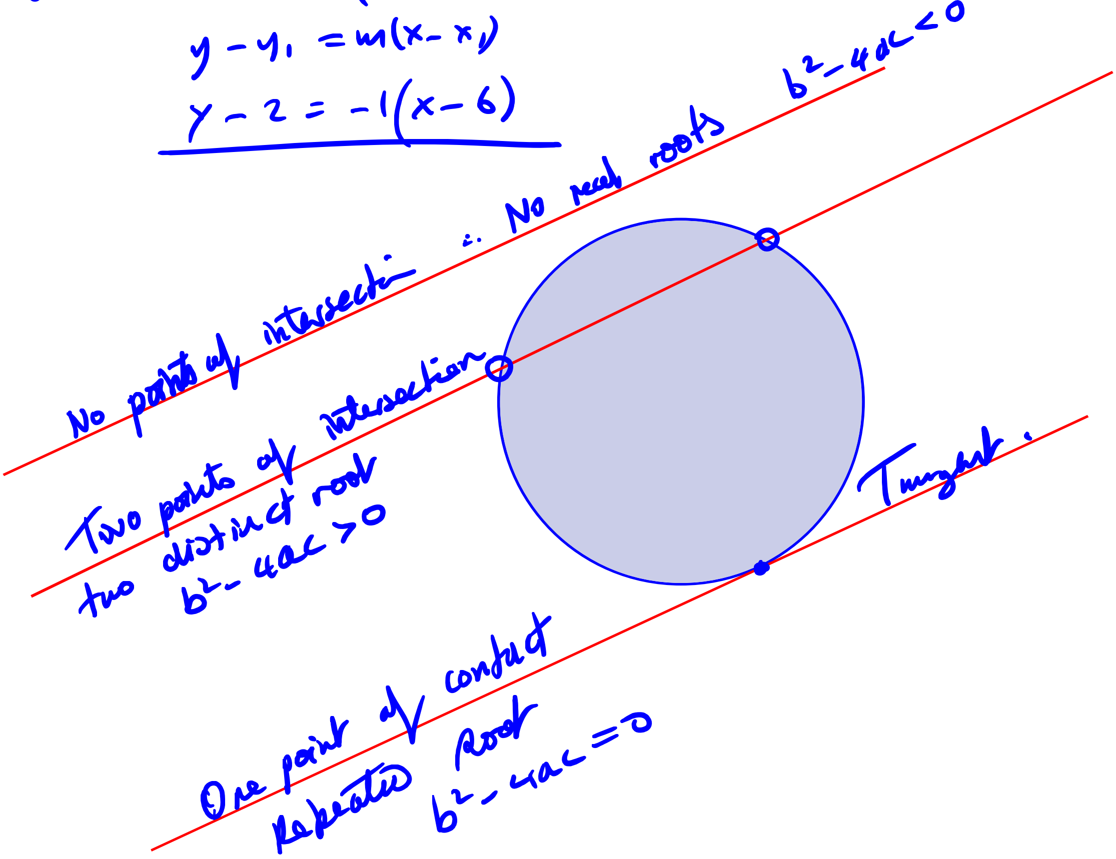
Circle Centre $(3,5)$ radius $\sqrt{18}=\underline{3\sqrt{2}}$

(b) Grad of OA $= \dfrac{5-2}{3-6} = \dfrac{3}{-3} = -1$ Gradient of Normal.

(c) Eqn of Normal $m=-1$ $A(6,2)$

$y-y_1=m(x-x_1)$

$\underline{y-2=-1(x-6)}$



(22) $2x+3y=k$ is a tangent to the circle $x^2+y^2+6x+4y=0$

$2x=k-3y$

$$4x^2+4y^2+24x+16y=0$$
$$(2x)^2+4y^2+12(2x)+16y=0$$
$$(k-3y)^2+4y^2+12(k-3y)+16y=0$$
$$k^2-6ky+9y^2+4y^2+12k-36y+16y=0$$
$$13y^2+\ -(6k+20)y+k^2+12k=0$$

the line $2x+3y=k$ is a tangent $\Rightarrow$ repeated root

$$\therefore\ b^2-4ac=0$$
$$(6k+20)^2-4(13)(k^2+12k)=0$$
$$(3k+10)^2-13(k^2+12k)=0$$
$$9k^2+60k+100-13k^2-156k=0$$
$$4k^2+96k-100=0$$
$$k^2+24k-25=0$$
$$(k+25)(k-1)=0\quad\Rightarrow\ \underline{k=1 \text{ or } k=-25}$$

WSB

29/06/20

Q2) $A(-5,6)$ $B(3,8)$

(a) Centre of circle at midpoint $\left(\frac{-5+3}{2}, \frac{6+8}{2}\right)=\left(\frac{-2}{2},\frac{14}{2}\right)=\underline{\underline{(-1,7)}}$

(b) Eqn of circle $(x-a)^2+(y-b)^2=r^2$

$(x+1)^2+(y-7)^2=r^2$

Use $B(3,8)$; $(3+1)^2+(8-7)^2=r^2$

$16+1=r^2 \Rightarrow \underline{r=\sqrt{17}}$

Eqn of circle $\underline{(x+1)^2+(y-7)^2=17}$

(c) Grad of OA (radius) $=\frac{\Delta y}{\Delta x}=\frac{6-7}{-5-(-1)}=\frac{-1}{-4}=\frac{1}{4}$

$O(-1,7)$ $A(-5,6)$

Grad of tangent $=-4$

$m=-4$, $A(-5,6)$

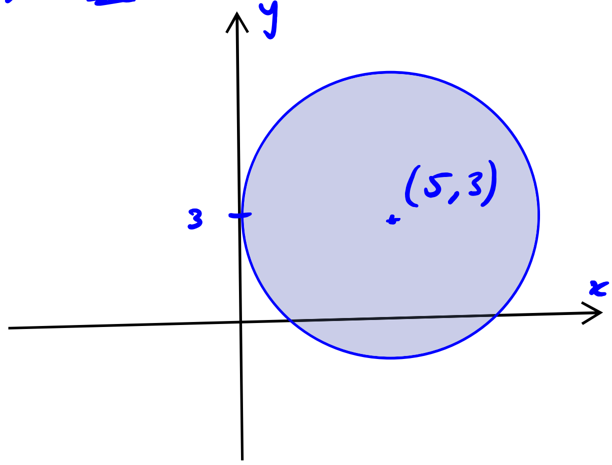
$y-y_1=m(x-x_1)$

$\underline{y-6=-4(x+5)}$

(5) $A(0,3)$ $\quad B(2,7)$

(a) grad of $AB = \frac{\Delta y}{\Delta x} = \frac{7-3}{2-0} = \frac{4}{2} = 2 \implies$ Grad of $\perp = -\frac{1}{2}$

Midpoint of $AB = \left(\frac{0+2}{2}, \frac{3+7}{2}\right) = \underline{(1,5)}$



eq$^{\underline{n}}$ of $\perp$ bisector

$m = -\frac{1}{2}$ $\quad (1,5)$

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{2}(x-1)$

(b) Centre of circle has $y=3$.

$\implies 3-5 = -\frac{1}{2}(x-1)$

$4 = x - 1 \implies x = 5$

Centre $(5,3)$

$(x-a)^2 + (y-b)^2 = r^2$

$(x-5)^2 + (y-3)^2 = r^2$

(x,y)

$A(0,3)$ $\quad 25 + (3-3)^2 = r^2 \implies r^2 = 25$

$\underline{\underline{r=5}}$

eq$^{\underline{n}}$ of circle $\underline{(x-5)^2 + (y-3)^2 = 25}$

(c) grad of radius OB $\quad O(5,3)$ $\quad B(2,7)$

$= \frac{\Delta y}{\Delta x} = \frac{7-3}{2-5} = \frac{4}{-3} = -\frac{4}{3}$

Grad of tangent $= \frac{3}{4}$ $\quad B(2,7)$

$y - y_1 = m(x - x_1)$

$y - 7 = \frac{3}{4}(x-2)$

$4y - 28 = 3x - 6$

$\underline{3x - 4y + 22 = 0}$

8

(a) $C_1: x^2+y^2-4y-16=0$

$x^2+(y-2)^2-4-16=0$

$x^2+(y-2)^2=20$

Circle centre $(0,2)$ radius $\sqrt{20}=2\sqrt{5}$

(b) $C_2: x^2+y^2-2x-8y-60=0$

$(x-1)^2+(y-4)^2-1-16-60=0$

$(x-1)^2+(y-4)^2=77$

$C_1(0,2)$ $C_2(1,4)$

Grad of line $=\frac{\Delta y}{\Delta x}=\frac{4-2}{1-0}=2$, $(0,2)$

$y=2x+2$

C_1 and the line $y=2x+2$ cross at two points one of which is P.

$x^2+(y-2)^2=20$

$x^2+(2x+2-2)^2=20$

$x^2+4x^2=20$

$5x^2=20$

$x^2=4$

$x=\pm 2$

At P $x=-2$ since P lies in the negative quadrant.

$x=-2 \Rightarrow y=-2$

$P(-2,-2)$

(c) grad of $l=-\frac{1}{2}$

$y-y_1=m(x-x_1)$

$y+2=-\frac{1}{2}(x+2)$

WSC

Q4 $A(5,5)$ $B(1,-7)$

(a) grad of OA $=\frac{\Delta y}{\Delta x}=\frac{5-0}{5-0}=\frac{5}{5}=1$

Grad of tangent $=-1$ $A(5,5)$

$y-y_1=-1(x-x_1)$

$y-5=-(x-5)$

(b) Grad of OB $=\frac{\Delta y}{\Delta x}=\frac{-7-0}{1-0}=\frac{-7}{1}=-7$

Grad of tangent $=\frac{1}{7}$ $B(1,-7)$

$y-y_1=m(x-x_1)$

$y+7=\frac{1}{7}(x-1)$

$7y+49=x-1$

$x-7y-50=0$

(c) Solve simultaneously

$y-5=-x+5$

$y=-x+10$

and $x-7y-50=0$

Sub

$\Rightarrow x-7(-x+10)-50=0$

$x+7x-70-50=0$

$8x=120$

$x=15$

$\Rightarrow y=-15+10=-5$

$C(15,-5)$

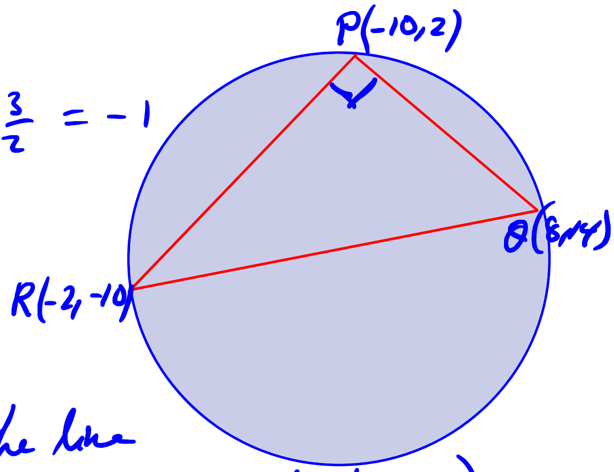
9. $P(-10,2)$ $Q(8,14)$ $R(-2,-10)$

(a) Grad of $PQ = \dfrac{14-2}{8-(-10)} = \dfrac{12}{18} = \dfrac{2}{3}$

Grad of $PR = \dfrac{-10-2}{-2-(-10)} = \dfrac{-12}{8} = -\dfrac{3}{2}$

Grad of PQ $\times$ Grad of $PR = \dfrac{2}{3} \times -\dfrac{3}{2} = -1$

$\therefore$ PR is $\perp$ to PQ.



we have a right angle at P

$\angle RPQ = 90^\circ$ $\therefore$ using circle theorem that angle in a semi circle is $90^\circ \Rightarrow$ the line QR is a diameter and the centre of the circle lies at the midpoint.

Midpoint of RQ $\left(\dfrac{-2+8}{2}, \dfrac{14-10}{2}\right) = \underline{\underline{(3,2)}}$

Equation of the circle $(x-3)^2 + (y-2)^2 = r^2$

Use $Q(8,14) \Rightarrow (8-3)^2 + (14-2)^2 = r^2$

$25 + 144 = r^2$

$169 = r^2 \Rightarrow \underline{\underline{r = 13}}$

$\therefore$ Eqn of circle $(x-3)^2 + (y-2)^2 = 169$

Expand $x^2 - 6x + 9 + y^2 - 4y + 4 - 169 = 0$

$\underline{x^2 + y^2 - 6x - 4y - 156 = 0}$ $\underline{\text{As required}}$

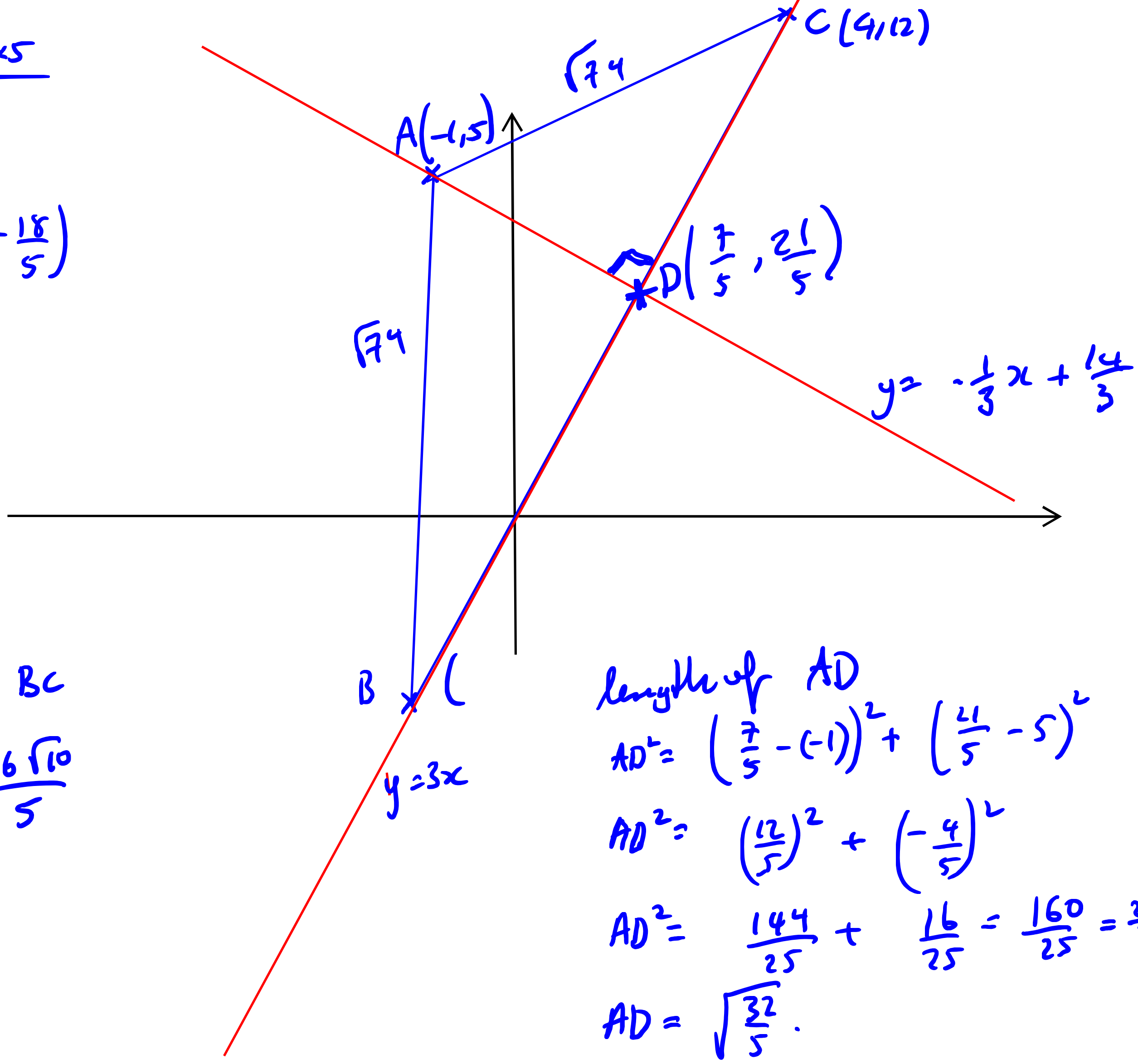
Mixed Ex5

Q17)

$A(-1,5)$

$B\left(-\frac{6}{5}, -\frac{18}{5}\right)$

$C(4,12)$



length of BC

$$BC^2 = \frac{26\sqrt{10}}{5}$$

length of AD

$$AD^2 = \left(\frac{7}{5} - (-1)\right)^2 + \left(\frac{21}{5} - 5\right)^2$$

$$AD^2 = \left(\frac{12}{5}\right)^2 + \left(-\frac{4}{5}\right)^2$$

$$AD^2 = \frac{144}{25} + \frac{16}{25} = \frac{160}{25} = \frac{32}{5}$$

$$AD = \sqrt{\frac{32}{5}}.$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times \frac{26\sqrt{10}}{5} \times \frac{\sqrt{32}}{\sqrt{5}}$$

$$= \frac{1}{2} \times \frac{26\cancel{\sqrt{5}}\sqrt{2}}{5} \times \frac{4\sqrt{2}}{\cancel{\sqrt{5}}}$$

$$= \frac{104}{5} \text{ units}^2$$

=

Mixed Ex 6

Q16 Centre $(6, 5)$ $(0, 12)$

$y = mx + 12$ equation of l_1 and l_2

$(x-6)^2 + (y-5)^2 = 17$

$(x-6)^2 + (mx + 12 - 5)^2 = 17$

$x^2 - 12x + 36 + m^2x^2 + 14mx + 49 = 17$

$(m^2+1)x^2 + (14m - 12)x + 68 = 0$

equation has a repeated root. (as lines l_1 and l_2 are tangent)

$b^2 - 4ac = 0$

$(14m - 12)^2 - 4(m^2+1)68 = 0$

$(7m - 6)^2 - 68(m^2+1) = 0$

$49m^2 - 84m + 36 - 68m^2 - 68 = 0$

$-19m^2 - 84m - 32 = 0$

$19m^2 + 84m + 32 = 0$

$(19m + 8)(m + 4) = 0$ 1 4 / 19 8

$m = -\frac{8}{19}$ or $m = -4$

$y = -4x + 12$ l_1

$y = -\frac{8}{19}x + 2$ l_2.