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|-------------------------------------------------------------------------------------------|--|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Please check the examination details below before entering your candidate information     |  |                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                 |
| Candidate surname                                                                         |  | Other names                                                                                                                                                                                                                     |                                                                                                                                                                                                                                 |
| <b>Pearson Edexcel</b><br><b>Level 3 GCE</b>                                              |  | Centre Number                                                                                                                                                                                                                   | Candidate Number                                                                                                                                                                                                                |
|                                                                                           |  | <input style="width: 20px; height: 20px;" type="text"/> <input style="width: 20px; height: 20px;" type="text"/> <input style="width: 20px; height: 20px;" type="text"/> <input style="width: 20px; height: 20px;" type="text"/> | <input style="width: 20px; height: 20px;" type="text"/> <input style="width: 20px; height: 20px;" type="text"/> <input style="width: 20px; height: 20px;" type="text"/> <input style="width: 20px; height: 20px;" type="text"/> |
| <b>Wednesday 14 October 2020</b>                                                          |  |                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                 |
| Afternoon (Time: 2 hours)                                                                 |  | Paper Reference <b>9MA0/02</b>                                                                                                                                                                                                  |                                                                                                                                                                                                                                 |
| <b>Mathematics</b><br><b>Advanced</b><br><b>Paper 2: Pure Mathematics 2</b>               |  |                                                                                                                                                                                                                                 |                                                                                                                                                                                                                                 |
| <b>You must have:</b><br>Mathematical Formulae and Statistical Tables (Green), calculator |  |                                                                                                                                                                                                                                 | Total Marks                                                                                                                                                                                                                     |

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ►

- 1 The table below shows corresponding values of  $x$  and  $y$  for  $y = \sqrt{\frac{x}{1+x}}$

The values of  $y$  are given to 4 significant figures.

|     |        |        |        |        |        |
|-----|--------|--------|--------|--------|--------|
| $x$ | 0.5    | 1      | 1.5    | 2      | 2.5    |
| $y$ | 0.5774 | 0.7071 | 0.7746 | 0.8165 | 0.8452 |

- (a) Use the trapezium rule, with all the values of  $y$  in the table, to find an estimate for

$$\int_{0.5}^{2.5} \sqrt{\frac{x}{1+x}} \, dx$$

giving your answer to 3 significant figures.

(3)

- (b) Using your answer to part (a), deduce an estimate for  $\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} \, dx$

(1)

Given that

$$\int_{0.5}^{2.5} \sqrt{\frac{9x}{1+x}} \, dx = 4.535 \text{ to 4 significant figures}$$

- (c) comment on the accuracy of your answer to part (b).

(1)

(Total for Question 1 is 5 marks)

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2. Relative to a fixed origin, points  $P$ ,  $Q$  and  $R$  have position vectors  $\mathbf{p}$ ,  $\mathbf{q}$  and  $\mathbf{r}$  respectively. Given that

- $P$ ,  $Q$  and  $R$  lie on a straight line
- $Q$  lies one third of the way from  $P$  to  $R$

show that

$$\mathbf{q} = \frac{1}{3}(\mathbf{r} + 2\mathbf{p})$$

(3)

(Total for Question 2 is 3 marks)

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3. (a) Given that

$$2 \log (4 - x) = \log (x + 8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

- (b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

- (ii) State which of the roots in (b)(i) is **not** a solution of

$$2 \log (4 - x) = \log (x + 8)$$

giving a reason for your answer.

(2)

(Total for Question 3 is 5 marks)

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4. In the binomial expansion of

$$(a + 2x)^7 \quad \text{where } a \text{ is a constant}$$

the coefficient of  $x^4$  is 15 120

Find the value of  $a$ .

(3)

(Total for Question 4 is 3 marks)

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5. The curve with equation  $y = 3 \times 2^x$  meets the curve with equation  $y = 15 - 2^{x+1}$  at the point  $P$

Find, using algebra, the exact  $x$  coordinate of  $P$ .

(4)

(Total for Question 5 is 4 marks)

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6. (a) Given that

$$\frac{x^2 + 8x - 3}{x + 2} \equiv Ax + B + \frac{C}{x + 2} \quad x \in \mathbb{R}, x \neq -2$$

find the values of the constants  $A$ ,  $B$  and  $C$

(3)

(b) Hence, using algebraic integration, find the exact value of

$$\int_0^6 \frac{x^2 + 8x - 3}{x + 2} \, dx$$

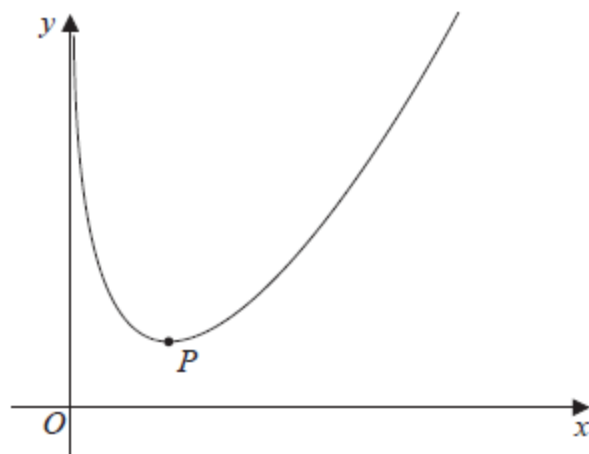
giving your answer in the form  $a + b \ln 2$  where  $a$  and  $b$  are integers to be found.

(4)

(Total for Question 6 is 7 marks)

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7.



**Figure 1**

Figure 1 shows a sketch of the curve  $C$  with equation

$$y = \frac{4x^2 + x}{2\sqrt{x}} - 4 \ln x \quad x > 0$$

(a) Show that

$$\frac{dy}{dx} = \frac{12x^2 + x - 16\sqrt{x}}{4x\sqrt{x}}$$

**(4)**

The point  $P$ , shown in Figure 1, is the minimum turning point on  $C$ .

(b) Show that the  $x$  coordinate of  $P$  is a solution of

$$x = \left( \frac{4}{3} - \frac{\sqrt{x}}{12} \right)^{\frac{2}{3}}$$

**(3)**

(c) Use the iteration formula

$$x_{n+1} = \left( \frac{4}{3} - \frac{\sqrt{x_n}}{12} \right)^{\frac{2}{3}} \quad x_1 = 2$$

- to find (i) the value of  $x_2$  to 5 decimal places,  
(ii) the  $x$  coordinate of  $P$  to 5 decimal places.

**(3)**

**(Total for Question 7 is 10 marks)**

8. A curve  $C$  has equation  $y = f(x)$

Given that

- $f'(x) = 6x^2 + ax - 23$  where  $a$  is a constant
- the  $y$  intercept of  $C$  is  $-12$
- $(x + 4)$  is a factor of  $f(x)$

find, in simplest form,  $f(x)$

(6)

(Total for Question 8 is 6 marks)

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9. A quantity of ethanol was heated until it reached boiling point.  
The temperature of the ethanol,  $\theta$  °C, at time  $t$  seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where  $A$  and  $B$  are positive constants.

Given that

- the initial temperature of the ethanol was 18 °C
- after 10 seconds the temperature of the ethanol was 44 °C

- (a) find a complete equation for the model, giving the values of  $A$  and  $B$  to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78 °C

- (b) Use this information to evaluate the model.

(2)

(Total for Question 9 is 6 marks)

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**10. In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that

$$\cos 3A \equiv 4 \cos^3 A - 3 \cos A \quad (4)$$

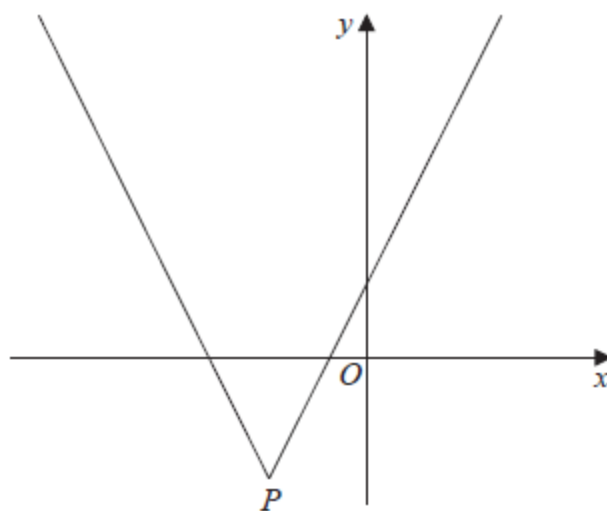
(b) Hence solve, for  $-90^\circ \leq x \leq 180^\circ$ , the equation

$$1 - \cos 3x = \sin^2 x \quad (4)$$

**(Total for Question 10 is 8 marks)**

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11.



**Figure 2**

Figure 2 shows a sketch of the graph with equation

$$y = 2|x + 4| - 5$$

The vertex of the graph is at the point  $P$ , shown in Figure 2.

(a) Find the coordinates of  $P$ .

(2)

(b) Solve the equation

$$3x + 40 = 2|x + 4| - 5$$

(2)

A line  $l$  has equation  $y = ax$ , where  $a$  is a constant.

Given that  $l$  intersects  $y = 2|x + 4| - 5$  at least once,

(c) find the range of possible values of  $a$ , writing your answer in set notation.

(3)

**(Total for Question 11 is 7 marks)**

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12.

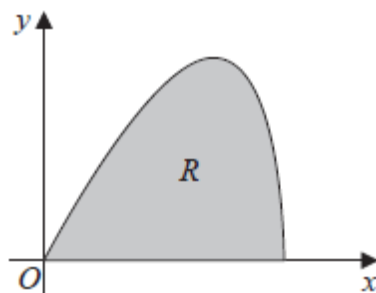


Figure 3

The curve shown in Figure 3 has parametric equations

$$x = 6 \sin t \quad y = 5 \sin 2t \quad 0 \leq t \leq \frac{\pi}{2}$$

The region  $R$ , shown shaded in Figure 3, is bounded by the curve and the  $x$ -axis.

- (a) (i) Show that the area of  $R$  is given by  $\int_0^{\frac{\pi}{2}} 60 \sin t \cos^2 t \, dt$  (3)
- (ii) Hence show, by algebraic integration, that the area of  $R$  is exactly 20 (3)

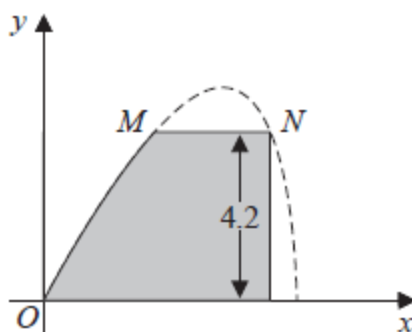


Figure 4

Part of the curve is used to model the profile of a small dam, shown shaded in Figure 4. Using the model and given that

- $x$  and  $y$  are in metres
- the vertical wall of the dam is 4.2 metres high
- there is a horizontal walkway of width  $MN$  along the top of the dam

- (b) calculate the width of the walkway. (5)

(Total for Question 12 is 11 marks)

13. The function  $g$  is defined by

$$g(x) = \frac{3 \ln(x) - 7}{\ln(x) - 2} \quad x > 0 \quad x \neq k$$

where  $k$  is a constant.

(a) Deduce the value of  $k$ .

(1)

(b) Prove that

$$g'(x) > 0$$

for all values of  $x$  in the domain of  $g$ .

(3)

(c) Find the range of values of  $a$  for which

$$g(a) > 0$$

(2)

(Total for Question 13 is 6 marks)

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14. A circle  $C$  with radius  $r$

- lies only in the 1st quadrant
- touches the  $x$ -axis and touches the  $y$ -axis

The line  $l$  has equation  $2x + y = 12$

(a) Show that the  $x$  coordinates of the points of intersection of  $l$  with  $C$  satisfy

$$5x^2 + (2r - 48)x + (r^2 - 24r + 144) = 0$$

(3)

Given also that  $l$  is a tangent to  $C$ ,

(b) find the two possible values of  $r$ , giving your answers as fully simplified surds.

(4)

(Total for Question 14 is 7 marks)

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15.

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A geometric series has common ratio  $r$  and first term  $a$ .

Given  $r \neq 1$  and  $a \neq 0$

(a) prove that

$$S_n = \frac{a(1-r^n)}{1-r} \quad (4)$$

Given also that  $S_{10}$  is four times  $S_5$

(b) find the exact value of  $r$ . (4)

**(Total for Question 15 is 8 marks)**

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16. Use algebra to prove that the square of any natural number is **either** a multiple of 3 **or** one more than a multiple of 3 (4)

**(Total for Question 16 is 4 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**