Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Mock Paper – Winter 2019

(Time: 2 hours) | Paper Reference **9MA0/02**

Mathematics

Advanced

Paper 2: Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

Use **black** ink or ball-point pen.
If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
Fill in the boxes at the top of this page with your name, centre number and candidate number.
Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
Answer the questions.
You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
Inexact answers should be given to three significant figures unless otherwise stated.

Information

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
There are 15 questions in this question paper. The total mark for this paper is 100.
The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question*.

Advice

Read each question carefully before you start to answer it.
Try to answer every question.
Check your answers if you have time at the end.

1. (a) Given that θ is small and in radians, show that the equation

$$\cos\theta - \sin\frac{1}{2}\theta + 2\tan\theta = \frac{11}{10} \qquad \text{(I)}$$

can be written as $5\theta^2 - 15\theta + 1 \approx 0$.

(3)

The solutions of the equation $5\theta^2 - 15\theta + 1 = 0$ are 0.068 and 2.932, correct to 3 decimal places.

(b) Comment on the validity of each of these values as approximate solutions to equation (I).

(1)

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2. A curve has parametric equations

$$x = 6t + 1, \quad y = 5 - \frac{4}{3t}, \qquad t \neq 0.$$

Show that the Cartesian equation of the curve can be expressed in the form

$$y = \frac{ax + b}{x - 1}, \quad x \neq k,$$

where a, b and k are constants to be found.

(3)

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3. A curve has equation $y = x^2 + kx + 14 - \dfrac{8}{(x-5)}$, where k is a constant.

Given that the curve has a stationary point P, where $x = 3$,

(a) show that $k = -8$.

(4)

(b) Determine the nature of the stationary point P, giving a reason for your answer.

(2)

(c) Show that the curve has a point of inflection where $x = 7$.

(2)

The curve passes through the points (4.5, 14.25) and (5.5, –15.75).

Jane uses this information to write down the following

Since there is a change of sign, the curve cuts the x-axis in the interval (4.5, 5.5).

(d) Explain the error in Jane's reasoning.

(1)

4.

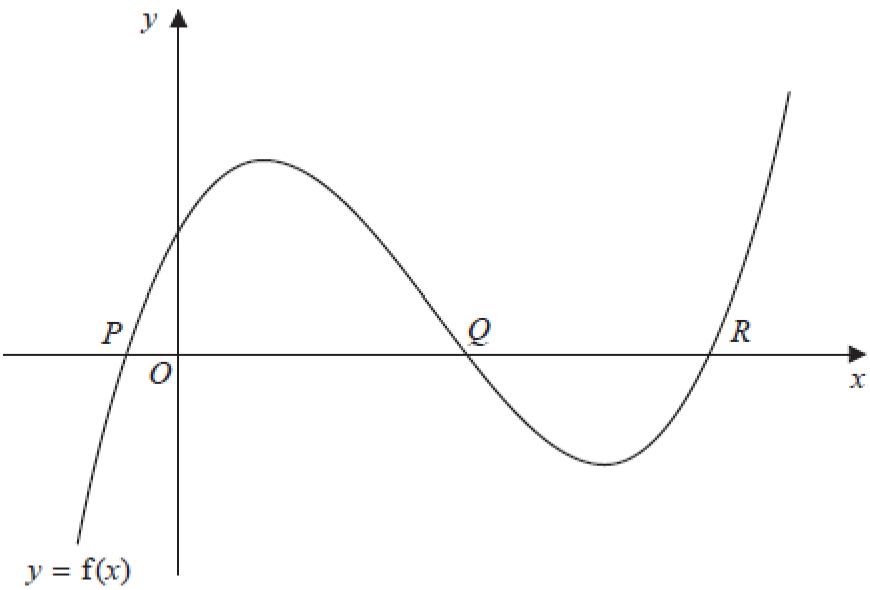

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x^3 - 6x^2 + 7x + 2, \quad x \in \mathbb{R}.$$

The curve cuts the x-axis at the points P, Q and R, as shown in Figure 1. The coordinates of Q are (2, 0).

(a) Write $\mathrm{f}(x)$ as a product of two algebraic factors.

(2)

(b) Find, giving your answer in simplest form,

(i) the exact x coordinate of P,

(ii) the exact x coordinate of R.

(2)

(c) Deduce the number of real solutions, for $-\pi \leq \theta \leq 12\pi$, to the equation

$$\sin^3 \theta - 6 \sin^2 \theta + 7 \sin \theta + 2 = 0,$$

justifying your answer.

(2)

5.

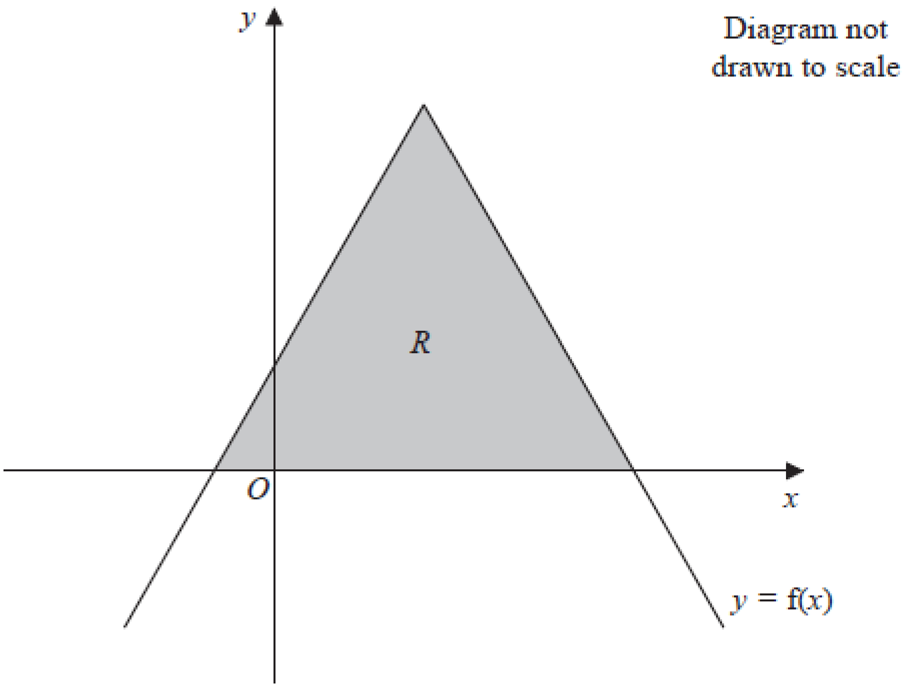

Figure 2

Figure 2 shows part of a graph with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 7 - |3x - 5|, \qquad x \in \mathbb{R}.$$

The finite region R, shown shaded in Figure 2, is bounded by the graph with equation $y = \mathrm{f}(x)$ and the x-axis.

(a) Find the area of R, giving your answer in simplest form.

(4)

The equation $7 - |3x - 5| = k$, where k is a constant, has two distinct real solutions.

(b) Write down the range of possible values for k.

(1)

6. $$\mathrm{f}(x) = (2 + kx)^{-4}, \text{ where } k \text{ is a positive constant.}$$

The binomial expansion of f(x), in ascending powers of x, up to and including the term in x^2, is

$$\frac{1}{16} + Ax + \frac{125}{32}x^2,$$

where A is a constant.

(a) Find the value of A, giving your answer in simplest form.

(5)

(b) Determine, giving a reason for your answer, whether the binomial expansion for f(x) is valid when $x = \dfrac{1}{10}$.

(1)

7.

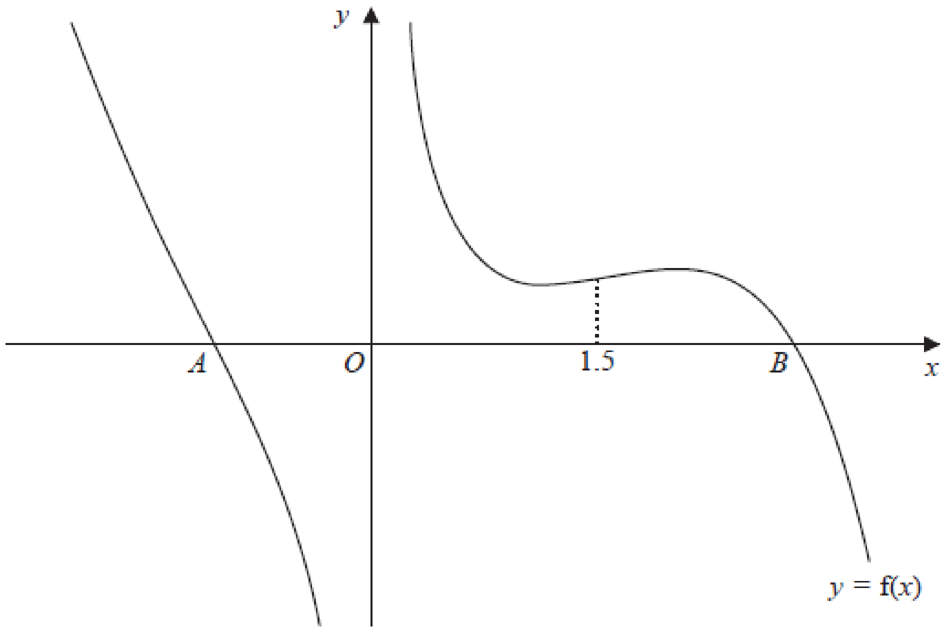

Figure 3

Figure 3 shows a plot of part of the curve with equation $y = \text{f}(x)$, where

$$\text{f}(x) = \frac{2}{x} - \text{e}^x + 2x^2, \quad x \in \mathbb{R}, \quad x \neq 0.$$

The curve cuts the x-axis at the point A, where $x = \alpha$, and at the point B, where $x = \beta$, as shown in Figure 3.

(a) Show that α lies between –1.5 and –1.

(2)

(b) The iterative formula $x_{n+1} = -\sqrt{\frac{1}{2}\text{e}^{x_n} - \frac{1}{x_n}}$, $n \in \mathbb{N}$, with $x_1 = -1$, can be used to estimate the value of α.

(i) Find the value of x_3 to 4 decimal places.

(ii) Find the value of α correct to 2 decimal places.

The value of β lies in the interval [1.5, 3]. A student takes 3 as her first approximation to β. Given $\text{f}(3) = -1.4189$ and $\text{f}'(3) = -8.3078$ to 4 decimal places,

(c) apply the Newton-Raphson method once to $\text{f}(x)$ to obtain a second approximation to β. Give your answer to 2 decimal places.

(2)

A different student takes a starting value of 1.5 as his first approximation to β.

(d) Use Figure 3 to explain whether or not the Newton-Raphson method with this starting value gives a good second approximation to β.

(2)

8.

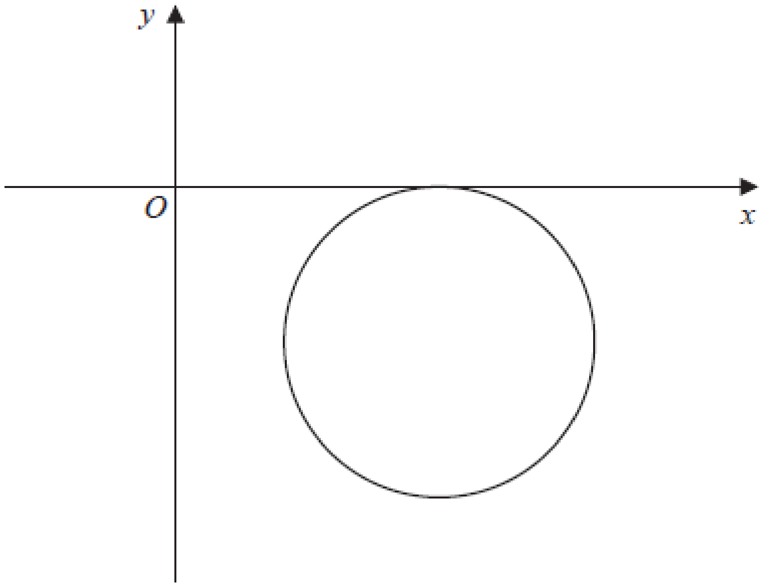

Figure 4

A circle with centre (9, –6) touches the x-axis as shown in Figure 4.

(a) Write down an equation for the circle.

(3)

A line l is parallel to the x-axis. The line l cuts the circle at points P and Q.

Given that the distance PQ is 8,

(b) find the two possible equations for l.

(4)

9. The amount of antibiotic, y milligrams, in a patient's bloodstream, t hours after the antibiotic was first given, is modelled by the equation $y = ab^t$, where a and b are constants.

(a) Show that this equation can be written in the form

$$\log_{10} y = t \log_{10} b + c,$$

expressing the constant c in terms of a.

(2)

A doctor measures the amount of antibiotic in the patient's bloodstream at regular intervals for the first 5 hours after the antibiotic was first given.

She plots a graph of $\log_{10} y$ against t and finds that the points on the graph lie close to a straight line passing through the point (0, 2.23) with gradient –0.076.

(b) Estimate, to 2 significant figures, the value of a and the value of b.

(2)

With reference to this model,

(c) (i) give a practical interpretation of the value of the constant a,

(ii) give a practical interpretation of the value of the constant b.

(2)

(d) Use the model to estimate the time taken, after the antibiotic was first given, for the amount of antibiotic in the patient's bloodstream to fall to 30 milligrams. Give your answer, in hours, correct to one decimal place.

(2)

(e) Comment on the reliability of your estimate in part (d).

(1)

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10. A biologist conducted an experiment to investigate the growth of mould on a slice of bread. The biologist measured the surface area of bread, A cm^2, covered by mould at times, t days, after the start of the experiment.

Initially 9.00 cm^2 of the bread was covered by mould and 6 days later, 56.25 cm^2 of the bread was covered by mould.

In the biologist's model, the rate of increase of the surface area of bread covered by mould, at any time t days, is proportional to the square root of that area.

By forming and solving a differential equation,

(a) show that the biologist's model leads to the equation

(6)

The biologist's full set of results are shown in the table below.

t (days)	0	6	12	18	24	30
A (cm^2)	9.00	56.25	143.78	271.19	334.81	337.33

Table 1

Use the last four measurements from Table 1 to

(b) (i) evaluate the biologist's model,

(ii) suggest a possible explanation of the results.

(3)

11.

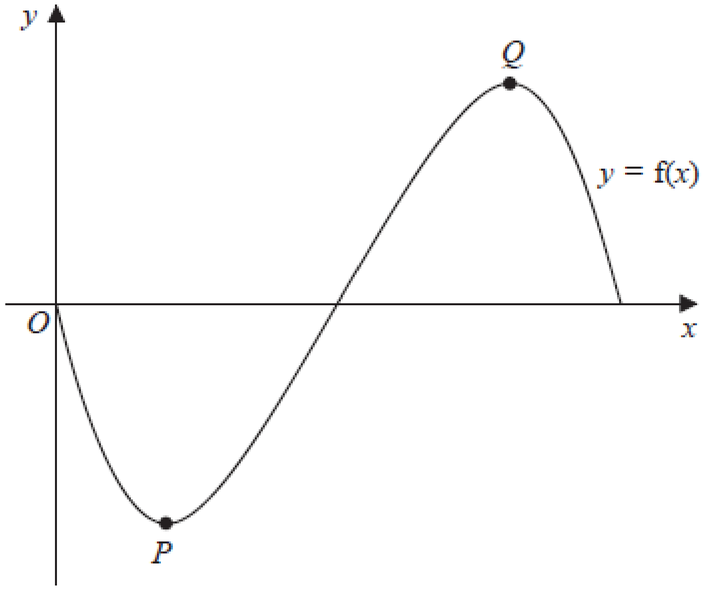

Figure 5

Figure 5 shows a sketch of the curve with equation $y = \text{f}(x)$, where

$$\text{f}(x) = \frac{\sin 2x}{-3 + \cos 2x}, \quad 0 \leq x \leq \pi.$$

The curve has a minimum turning point at P and a maximum turning point at Q, as shown in Figure 5.

(a) Show that the x coordinate of P and the x coordinate of Q are solutions of the equation

$$\cos 2x = \frac{1}{3}.$$

(4)

(b) Hence find, to 2 decimal places, the x coordinate of the maximum turning point on the curve with equation

(i) $y = \text{f}(3x) + 5, \qquad 0 \leq x \leq \frac{\pi}{3}$

(ii) $y = -\text{f}\left(\frac{1}{4}x\right), \qquad 0 \leq x \leq 4\pi.$

(4)

12. A company extracted 4500 tonnes of a mineral from a mine during 2018. The mass of the mineral which the company expects to extract in each subsequent year is modelled to decrease by 2% each year.

(a) Find the total mass of the mineral which the company expects to extract from 2018 to 2040 inclusive, giving your answer to 3 significant figures.

(2)

(b) Find the mass of the mineral which the company expects to extract during 2040, giving your answer to 3 significant figures.

(2)

The costs of extracting the mineral each year are assumed to be:

- £800 per tonne for the first 1500 tonnes,
- £600 per tonne for any amount in excess of 1500 tonnes.

The expected cost of extracting the mineral from 2018 to 2040 inclusive is £x million.

(c) Find the value of x, giving your answer to 3 significant figures.

(3)

13.

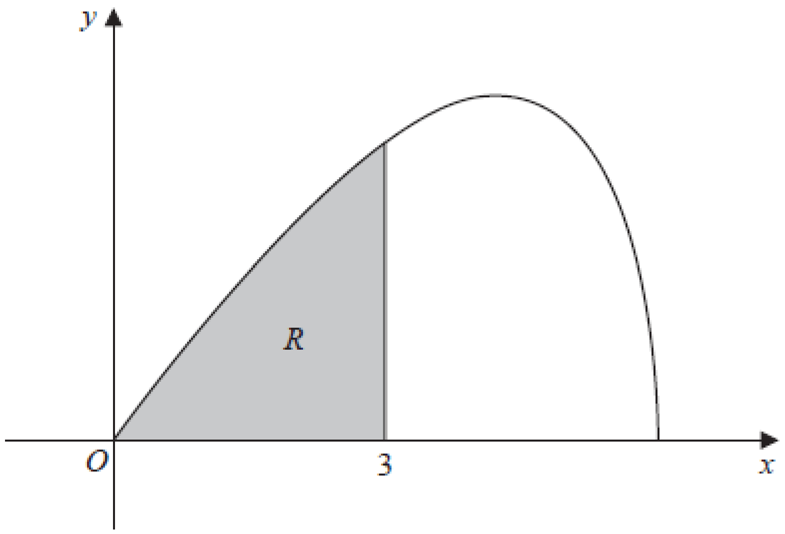

Figure 6

Figure 6 shows the curve with parametric equations

$$x = 6 \cos t, \qquad y\ 5 \sin 2t, \qquad 0 \leq t \leq \frac{\pi}{2}.$$

The finite region R, shown shaded in Figure 6, is bounded by the curve, the x-axis, and the line with equation $x = 3$.

Use calculus to show that the area of R is $20 - \frac{15}{2}\sqrt{3}$.

(7)

14.

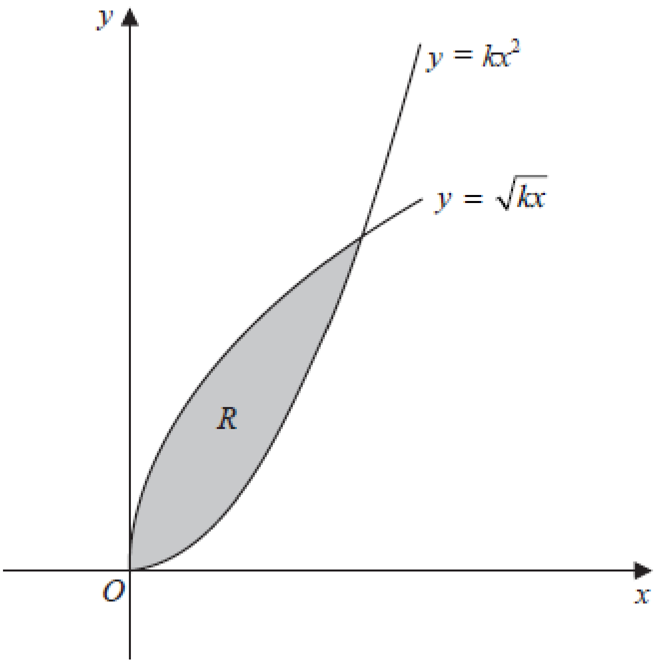

Figure 7

Figure 7 shows the curves with equations

$$y = kx^2, \qquad x \geq 0,$$

$$y = \sqrt{kx}, \qquad x \geq 0,$$

where k is a positive constant.

The finite region R, shown shaded in Figure 7, is bounded by the two curves.

Show that, for all values of k, the area of R is $\frac{1}{3}$.

(5)

15. A sequence of numbers a_1, a_2, a_3, … is defined by

$$a_{n+1} = k - \frac{3k}{a_n}, \qquad n \in \mathbb{Z}^+,$$

where k is a constant.

The sequence is periodic of order 3.

Given that $a_2 = 2$

(a) show that $k^2 + k - 12 = 0$.

(3)

Given that $a_1 \neq a_2$

(b) find the value of $\sum_{r=1}^{121} a_r$.

(4)

TOTAL FOR PAPER IS 100 MARKS