Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | | | | | |

Candidate Number | | | | |

Mock Paper – Winter 2019

(Time: 2 hours) Paper Reference **9MA0/01**

Mathematics

Advanced

Paper 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

Use **black** ink or ball-point pen.
If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
Fill in the boxes at the top of this page with your name, centre number and candidate number.
Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
Answer the questions.
You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
Inexact answers should be given to three significant figures unless otherwise stated.

Information

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
There are 15 questions in this question paper. The total mark for this paper is 100.
The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question*.

Advice

Read each question carefully before you start to answer it.
Try to answer every question.
Check your answers if you have time at the end.

1.

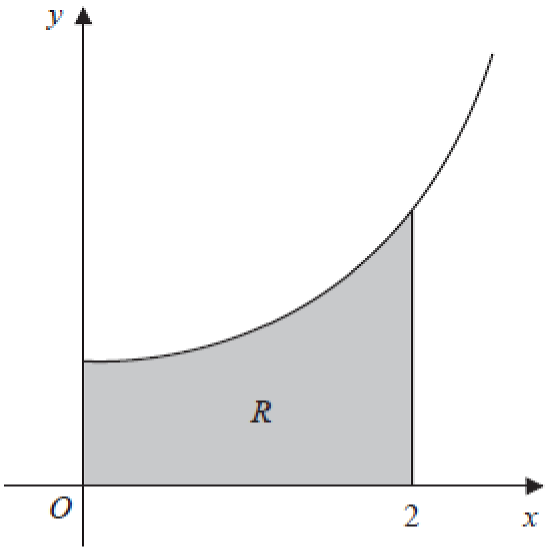

Figure 1

Figure 1 shows part of the curve with equation $y = e^{\frac{1}{5}x^2}$ for $x \geq 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the y-axis, the x-axis, and the line with equation $x = 2$.

The table below shows corresponding values of x and y for $y\ x = e^{\frac{1}{5}x^2}$.

x	0	0.5	1	1.5	2
y	1	$e^{0.05}$	$e^{0.2}$	$e^{0.45}$	$e^{0.8}$

(a) Use the trapezium rule, with all the values of y in the table, to find an estimate for the area of R, giving your answer to 2 decimal places.

(3)

(b) Use your answer to part (a) to deduce an estimate for

(i) $\displaystyle\int_0^2 (4 + e^{\frac{1}{5}x^2})\ dx$,

(ii) $\displaystyle\int e^{\frac{1}{5}(x-1)^2}\ dx$,

giving your answers to 2 decimal places.

(2)

2.

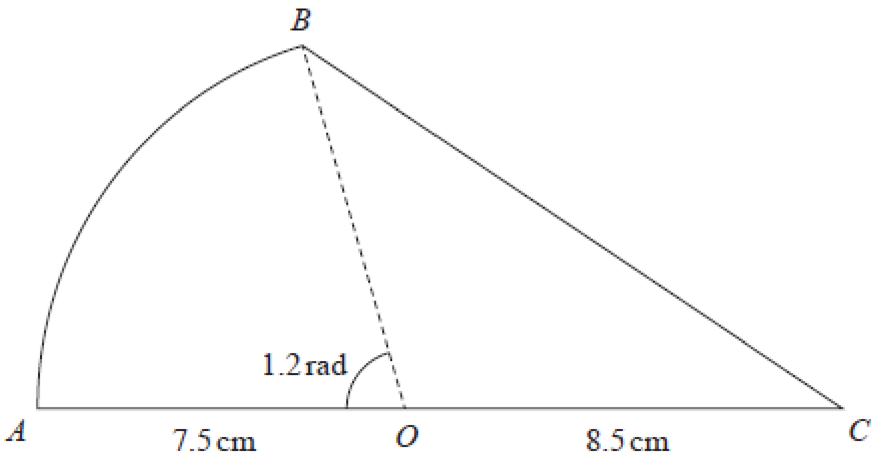

Figure 2

The shape *AOCBA*, shown in Figure 2, consists of a sector *AOB* of a circle centre *O* joined to a triangle *BOC*.

The points *A*, *O* and *C* lie on a straight line with $AO = 7.5$ cm and $OC = 8.5$ cm. The size of angle *AOB* is 1.2 radians.

Find, in cm, the perimeter of the shape *AOCBA*, giving your answer to one decimal place.

(5)

---

3. The equation $3x^2 + k = 5x + 2$, $k \in \mathbb{R}$, where k is a constant, has no real roots.

Find the range of possible values for k.

(4)

---

4. The function f is defined by $\quad \mathrm{f}(x) = \dfrac{12x}{3x+4}, \quad x \in \mathbb{R}, \quad x \geq 0.$

(a) Find the range of f.

(2)

(b) Find f^{-1}.

(3)

(c) Show, for $x \in \mathbb{R}$, $x \neq 0$, that $\mathrm{ff}(x) = \dfrac{9x}{3x+1}$.

(3)

(d) Show that $\mathrm{f\,f}(x) = \dfrac{7}{2}$ has no solutions.

(2)

---

5. A curve has equation $y = 4x^2 - 5x$. The curve passes through the point $P(2, 6)$

Use differentiation from first principles to find the value of the gradient of the curve at P.

(5)

6.

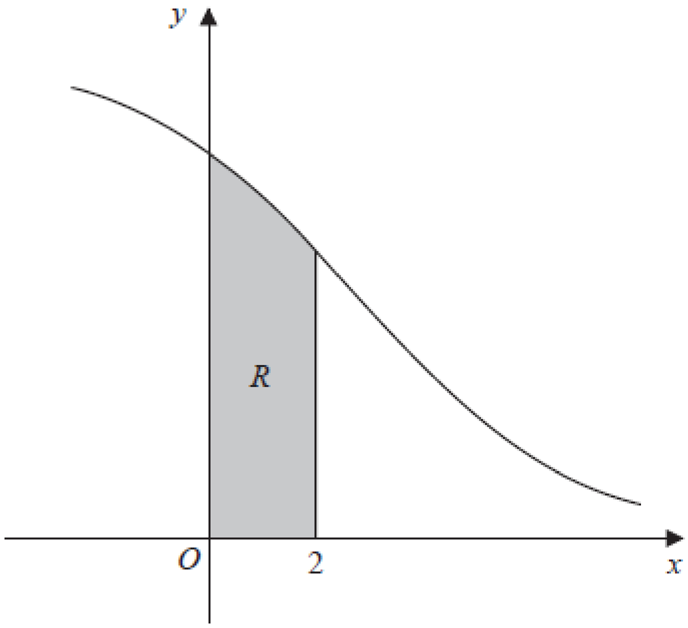

Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{6}{e^{\frac{1}{2}x} + 4}, \quad x \in \mathbb{R}.$$

The finite region R, shown shaded in Figure 3, is bounded by the curve, the y-axis, the x-axis, and the line with equation $x = 2$.

(a) Use the substitution $u = e^{\frac{1}{2}x}$ to show that the area of R can be given by

$$\int_a^b \frac{12}{u(u+4)}\,du,$$

where a and b are constants to be found.

(3)

(b) Hence use algebraic integration to show that the exact area of R is $3\ln\left(\frac{5e}{e+4}\right)$.

(5)

---

7. (a) Express $3 \sin \theta - 4 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$. State the value of R and give the value of α to 2 decimal places.

(3)

The temperature in a greenhouse, G °C, is modelled by the equation

$$G = 17 + 3 \sin (15t)° - 4 \cos (15t)°, \quad 0 \leq t \leq 17,$$

where t is the time in hours after 5 a.m.

(b) Find, according to this model,

(i) the maximum temperature in the greenhouse,

(1)

(ii) the time, after midday, when the temperature in the greenhouse is 20 °C. Give your answer to the nearest minute.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

---

8. (i) Show that $y^2 - 4y + 7$ is positive for all real values of y.

(2)

Bobby claims that

$$e^{3x} \geq e^{2x}, \quad x \in \mathbb{R}.$$

(ii) Determine whether Bobby's claim is always true, sometimes true or never true, justifying your answer.

(2)

Elsa claims that

'for $n \in \mathbb{Z}^+$, if n^2 is even, then n must be even'

(iii) Use proof by contradiction to show that Elsa's claim is true.

(2)

Ying claims that

'the sum of two different irrational numbers is irrational'

(iv) Determine whether Ying's claim is always true, sometimes true or never true, justifying your answer.

(2)

---

9. (a) Show that

$$\frac{\sin x}{1-\cos x} + \frac{1-\cos x}{\sin x} = k \operatorname{cosec} x, \qquad \equiv x \neq n\pi, \quad n \in \mathbb{Z},$$

where k is a constant to be found.

(4)

(b) Hence explain why the equation

$$\frac{\sin x}{1-\cos x} + \frac{1-\cos x}{\sin x} = 1.6$$

has no real solutions.

(1)

10.

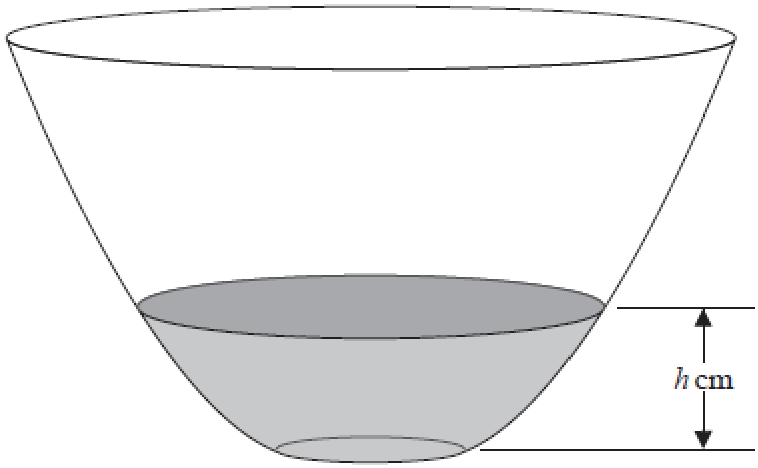

Figure 4

Figure 4 shows a bowl with a circular cross-section. Initially the bowl is empty. Water begins to flow into the bowl.

At time t seconds after the water begins to flow into the bowl, the height of the water in the bowl is h cm.

The volume of water, V cm^3, in the bowl is modelled as

$$V = 4\pi h(h + 6), \quad 0 \leq h \leq 25.$$

The water flows into the bowl at a constant rate of 80π cm3 s^{-1}.

(a) Show that, according to the model, it takes 36 seconds to fill the bowl with water from empty to a height of 24 cm.

(1)

(b) Find, according to the model, the rate of change of the height of the water, in cm s^{-1}, when $t = 8$.

(8)

11. Given that $y = a^x$, where a is a positive constant,

(i) show that

$$\frac{dy}{dx} = a^x \ln a.$$

(2)

Given that $x = 2 \tan y$, $-\frac{\pi}{2} < y < \frac{\pi}{2}$,

(ii) show that

$$\frac{dy}{dx} = \frac{k}{4 + x^2},$$

where k is a constant to be found.

(4)

12.

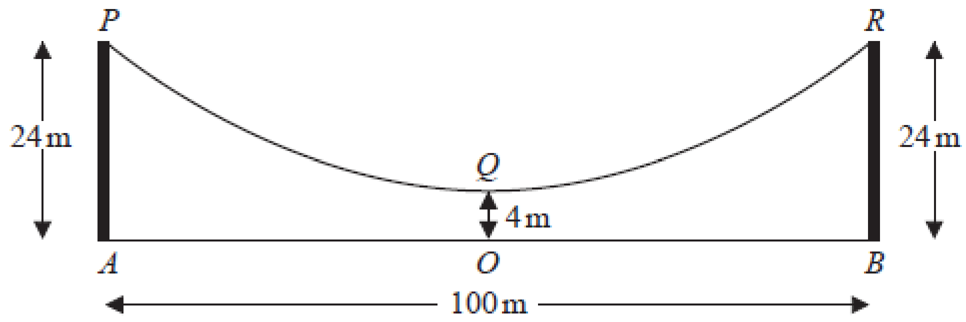

Figure 5

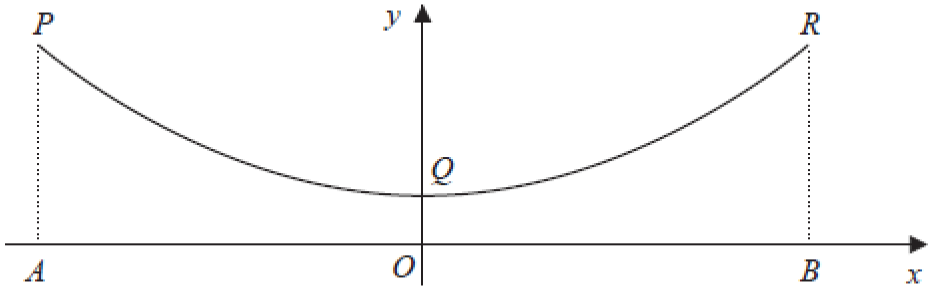

Figure 6

A suspension bridge cable *PQR* hangs between the tops of two vertical towers, *AP* and *BR*, as shown in Figure 5.

A walkway *AOB* runs between the bases of the towers, directly under the cable. The towers are 100 m apart and each tower is 24 m high.

At the point *O*, midway between the towers, the cable is 4 m above the walkway.

The points *P*, *Q*, *R*, *A*, *O* and *B* are assumed to lie in the same vertical plane and *AOB* is assumed to be horizontal.

Figure 6 shows a symmetric quadratic curve *PQR* used to model this cable.

Given that *O* is the origin,

(a) find an equation for the curve *PQR*.

(3)

Lee can safely inspect the cable up to a height of 12 m above the walkway. A defect is reported on the cable at a location 19 m horizontally **from one of the towers**.

(b) Determine whether, according to the model, Lee can safely inspect this defect.

(2)

(c) Give a reason why this model may not be suitable to determine whether Lee can safely inspect this defect.

(1)

13. Given that p is a positive constant,

(a) show that

$$\sum_{n=1}^{11} \ln(p^n) = k \ln p,$$

where k is a constant to be found,

(2)

(b) show that

$$\sum_{n=1}^{11} \ln(8p^n) = 33 \ln(2p^2).$$

(2)

(c) Hence find the set of values of p for which

$$\sum_{n=1}^{11} \ln(8p^n) = < 0,$$

giving your answer in set notation.

(2)

14.

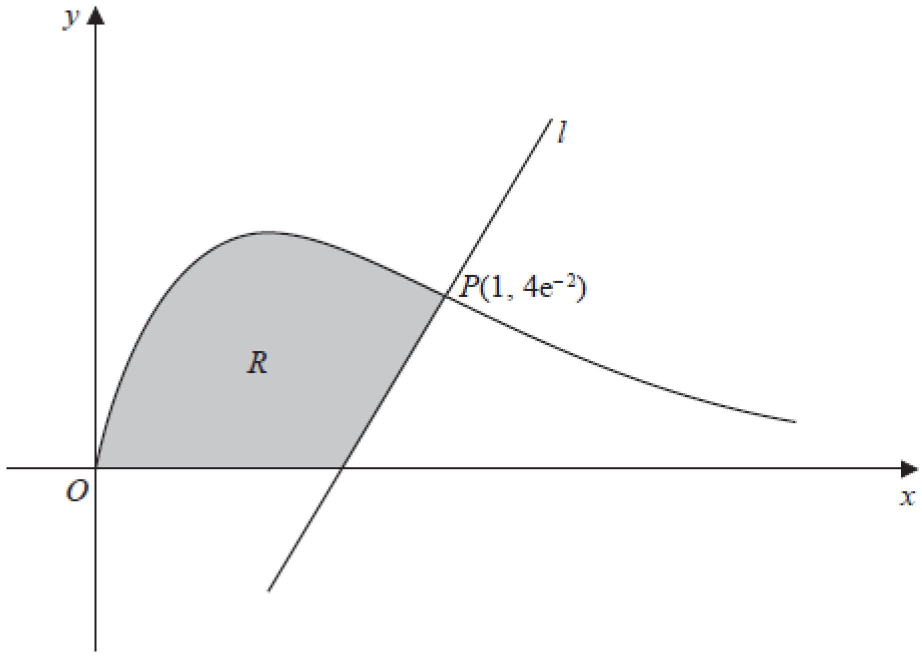

Figure 7

Figure 7 shows a sketch of the curve with equation $y = 4xe^{-2x}$, $x \geq 0$.

The line l is the normal to the curve at the point $P(1, 4e^{-2})$.The finite region R, shown shaded in Figure 7, is bounded by the curve, the line l, and the x-axis.

Find the exact value of the area of R.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(10)

15. Relative to a fixed origin O, the points A and B are such that

$$\overrightarrow{OA} = \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix} \text{ and } \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ p \end{pmatrix}, \text{ where } p \text{ is a constant,}$$

and the points C and D are such that

$$\overrightarrow{BC} = \begin{pmatrix} 0 \\ 6 \\ -7 \end{pmatrix} \text{ and } \overrightarrow{AD} = \begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix}.$$

(a) Find the position vector of the point D.

(1)

Given that $ABCD$ is a trapezium,

(b) find the value of p.

(4)

TOTAL FOR PAPER IS 100 MARKS