Pearson Edexcel Level 3

GCE Mathematics

Advanced

Paper 1: Pure Mathematics

Mock paper Spring 2018	**Paper Reference(s)**
Time: 2 hours	**9MA0/01**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this paper. The total mark is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Answer ALL questions.

1.

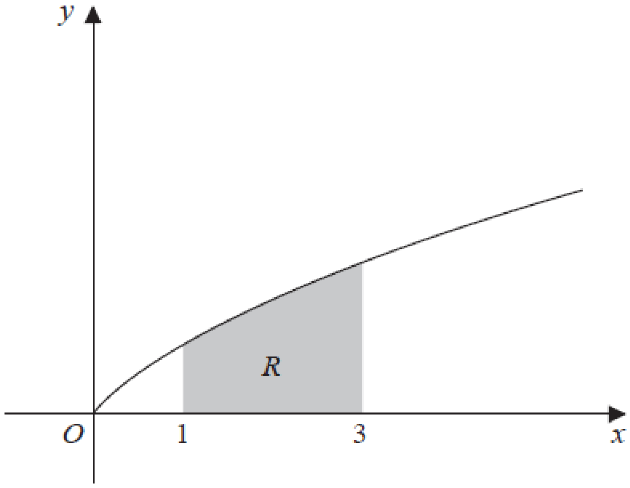

Figure 1

Figure 1 shows a sketch of the curve with equation $y = \dfrac{x}{1+\sqrt{x}}$, $x \geq 0$.

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = 1$, the x-axis and the line with equation $x = 3$.

The table below shows corresponding values of x and y for $y = \dfrac{x}{1+\sqrt{x}}$.

x	1	1.5	2	2.5	3
y	0.5	0.6742	0.8284	0.9686	1.0981

(*a*) Use the trapezium rule, with all the values of y in the table, to find an estimate for the area of R, giving your answer to 3 decimal places.

(3)

(*b*) Explain how the trapezium rule can be used to give a better approximation for the area of R.

(1)

(*c*) Giving your answer to 3 decimal places in each case, use your answer to part (a) to deduce an estimate for

(i) $\displaystyle\int_1^3 \frac{5x}{1+\sqrt{x}}\,dx$, (ii) $\displaystyle\int_1^3 6 + \frac{x}{1+\sqrt{x}}\,dx$.

(2)

(Total for Question 1 is 6 marks)

2. (*a*) Show that the binomial expansion of $(4+5x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^2 is

$$2 + \frac{5}{4}x + kx^2,$$

giving the value of the constant k as a simplified fraction.

(4)

(*b*) (i) Use the expansion from part (a), with $x = \frac{1}{10}$, to find an approximate value for $\sqrt{2}$.

Give your answer in the form $\frac{p}{q}$, where p and q are integers.

(ii) Explain why substituting $x = \frac{1}{10}$ into this binomial expansion leads to a valid approximation.

(4)

(Total for Question 2 is 8 marks)

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3. A sequence of numbers $a_1, a_2, a_3, \ldots,$ is defined by

$$a_1 = 3,$$

$$a_{n+1} = \frac{a_n - 3}{a_n - 2}, \quad n \in \mathbb{N}.$$

(*a*) Find $\sum_{r=1}^{100} a_r$.

(3)

(*b*) Hence find $\sum_{r=1}^{100} a_r + \sum_{r=1}^{99} a_r$

(1)

(Total for Question 3 is 4 marks)

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4. Relative to a fixed origin O,

the point A has position vector $\mathbf{i} + 7\mathbf{j} - 2\mathbf{k}$,
the point B has position vector $4\mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$,
and the point C has position vector $2\mathbf{i} + 10\mathbf{j} + 9\mathbf{k}$.

Given that $ABCD$ is a parallelogram,

(a) find the position vector of point D.

(2)

The vector $\overrightarrow{AX}$ has the same direction as $\overrightarrow{AB}$.

Given that $|\overrightarrow{AX}| = 10\sqrt{2}$,

(b) find the position vector of X.

(3)

(Total for Question 4 is 5 marks)

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5. $$f(x) = x^3 + ax^2 - ax + 48, \text{ where } a \text{ is a constant.}$$

Given that $f(-6) = 0$,

(a) (i) show that $a = 4$.

(ii) express $f(x)$ as a product of two algebraic factors.

(4)

Given that $2\log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3$,

(b) show that $x^3 + 4x^2 - 4x + 48 = 0$.

(4)

(c) Hence explain why $2\log_2(x + 2) + \log_2 x - \log_2(x - 6) = 3$ has no real roots.

(2)

(Total for Question 5 is 10 marks)

6.

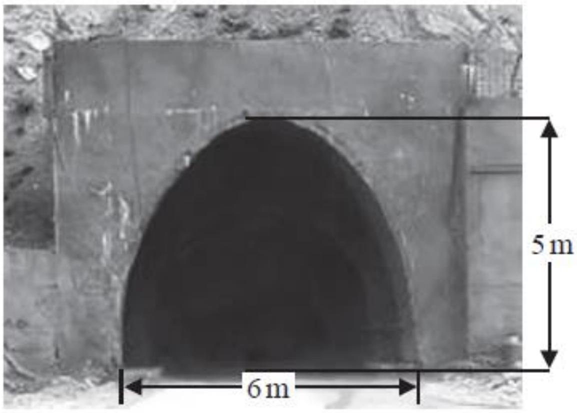

Figure 2

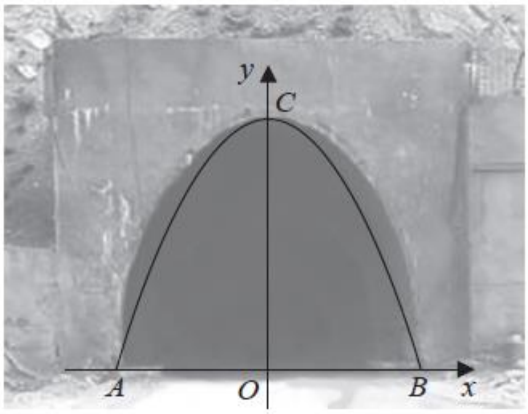

Figure 3

Figure 2 shows the entrance to a road tunnel. The maximum height of the tunnel is measured as 5 metres and the width of the base of the tunnel is measured as 6 metres.

Figure 3 shows a quadratic curve *BCA* used to model this entrance.

The points *A*, *O*, *B* and *C* are assumed to lie in the same vertical plane and the ground *AOB* is assumed to be horizontal.

(*a*) Find an equation for curve *BCA*.

(3)

A coach has height 4.1 m and width 2.4 m.

(*b*) Determine whether or not it is possible for the coach to enter the tunnel.

(2)

(*c*) Suggest a reason why this model may not be suitable to determine whether or not the coach can pass through the tunnel.

(1)

(Total for Question 6 is 6 marks)

7.

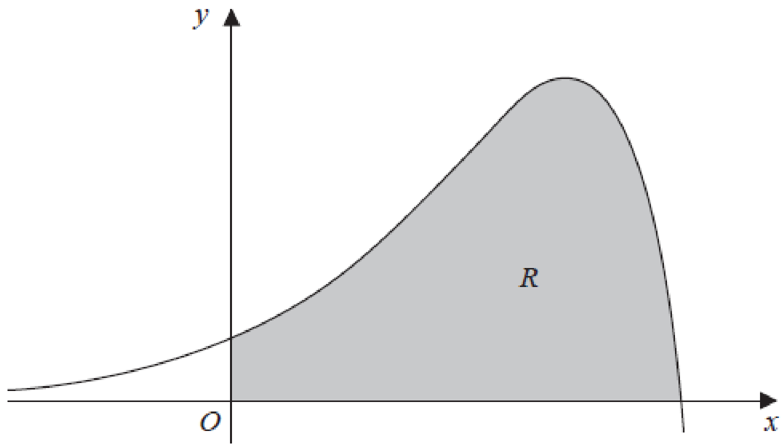

Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = 2e^{2x} - xe^{2x}, \quad x \in \mathbb{R}.$$

The finite region R, shown shaded in Figure 4, is bounded by the curve, the x-axis and the y-axis.

Use calculus to show that the exact area of R can be written in the form $pe^4 + q$, where p and q are rational constants to be found.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(Total for Question 7 is 5 marks)

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8. There were 2100 tonnes of wheat harvested on a farm during 2017.

The mass of wheat harvested during each subsequent year is expected to increase by 1.2% per year.

(*a*) Find the total mass of wheat expected to be harvested from 2017 to 2030 inclusive, giving your answer to 3 significant figures.

(2)

Each year it costs

- £5.15 per tonne to harvest the first 2000 tonnes of wheat
- £6.45 per tonne to harvest wheat in excess of 2000 tonnes

(*b*) Use this information to find the expected cost of harvesting the wheat from 2017 to 2030 inclusive. Give your answer to the nearest £1000.

(3)

(Total for Question 8 is 5 marks)

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9. The curve C has equation $y = 2x^3 + 5$.

The curve C passes through the point $P(1, 7)$.

Use differentiation from first principles to find the value of the gradient of the tangent to C at P.

(Total for Question 9 is 5 marks)

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10. The function f is defined by

$$\mathrm{f} : x \mapsto \frac{3x-5}{x+1}, \quad x \in \mathbb{R}, \quad x \neq -1.$$

(*a*) Find $\mathrm{f}^{-1}(x)$.

(3)

(*b*) Show that

$$\mathrm{ff}(x) = \frac{x+a}{x-1} \quad x \in \mathbb{R}, \quad x \neq -1,$$

where a is an integer to be found.

(4)

The function g is defined by

$$\mathrm{g} : x \mapsto x^2 - 3x, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5.$$

(*c*) Find the value of fg(2).

(2)

(*d*) Find the range of g.

(3)

(e) Explain why the function g does not have an inverse.

(1)

(Total for Question 10 is 13 marks)

11.

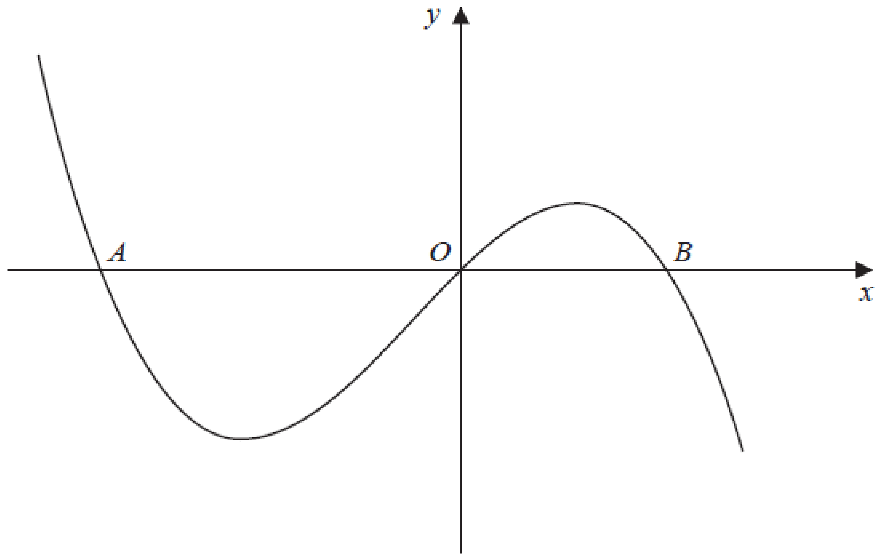

Figure 5

Figure 5 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$.

The curve C crosses the x-axis at the origin, O, and at the points A and B as shown in Figure 5.

Given that $\mathrm{f}'(x) = k - 4x - 3x^2$, where k is a constant,

(a) show that C has a point of inflection at $x = -\frac{2}{3}$.

(3)

Given also that the distance $AB = 4\sqrt{2}$,

(b) find, showing your working, the integer value of k.

(7)

(Total for Question 11 is 10 marks)

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12. Show that

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{1 + \cos\theta}\,\mathrm{d}\theta = 2 - 2\ln 2.$$

(Total for Question 12 is 7 marks)

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13. (*a*) Express $2 \sin \theta - 1.5 \cos \theta$ in the form $R \sin (\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$.

State the value of R and give the value of α to 4 decimal places.

(3)

Tom models the depth of water, D metres, at Southview harbour on 18th October 2017 by the formula

$$D = 6 + 2 \sin \left(\frac{4\pi t}{25}\right) - 1.5 \cos \left(\frac{4\pi t}{25}\right), \quad 0 \leq t \leq 24,$$

where t is the time, in hours, after 00:00 hours on 18th October 2017.

Use Tom's model to

(*b*) find the depth of water at 00:00 hours on 18th October 2017,

(1)

(*c*) find the maximum depth of water,

(1)

(*d*) find the time, in the afternoon, when the maximum depth of water occurs.
Give your answer to the nearest minute.

(3)

Tom's model is supported by measurements of D taken at regular intervals on 18th October 2017. Jolene attempts to use a similar model in order to model the depth of water at Southview harbour on 19th October 2017.

Jolene models the depth of water, H metres, at Southview harbour on 19th October 2017 by the formula

$$H = 6 + 2 \sin \left(\frac{4\pi x}{25}\right) - 1.5 \cos \left(\frac{4\pi x}{25}\right), \quad 0 \leq x \leq 24,$$

where x is the time, in hours, after 00:00 hours on 19th October 2017.

By considering the depth of water at 00:00 hours on 19th October 2017 for both models,

(*e*) (i) explain why Jolene's model is not correct,

(ii) hence find a suitable model for H in terms of x.

(3)

(Total for Question 13 is 11 marks)

14.

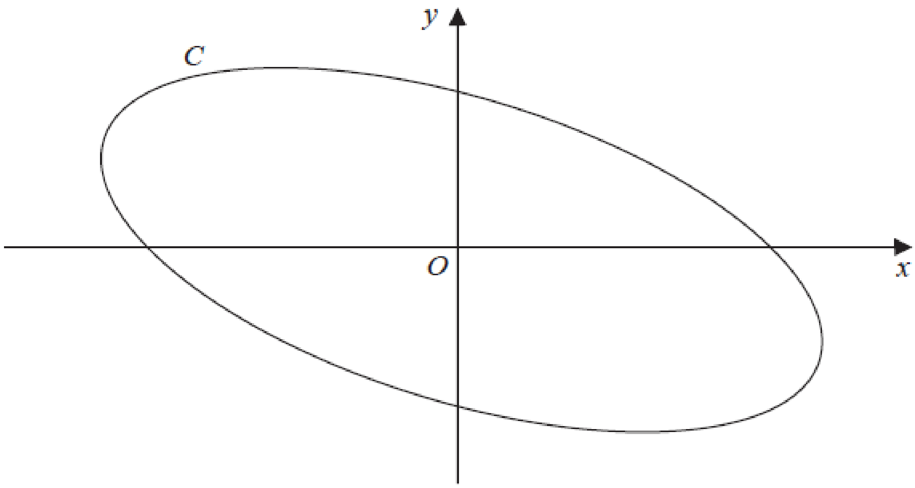

Figure 6

Figure 6 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t, \quad 0 < t \leq 2\pi.$$

Show that a Cartesian equation of C can be written in the form

$$(x + y)^2 + ay^2 = b,$$

where a and b are integers to be found.

(5)

(Total for Question 14 is 5 marks)

TOTAL FOR PAPER IS 100 MARKS